Azimuth and elevation angle estimation with no failure and no eigen decomposition

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Abstract

Recently, Wu et al. proposed a scheme for two-dimensional direction of arrival angle estimation for azimuth and elevation angles, using the propagator method. An advantage of this method over the classical subspace based algorithms, such as ESPRIT and MUSIC, is that it does not apply any eigenvalue decomposition (EVD) to the cross spectral matrix or singular value decomposition (SVD) to the received data. This significantly reduces the computational complexity, compared to the EVD and SVD. However, Wu's method has some drawbacks, such as pair matching between the azimuth and elevation angle estimations for multiple different sources. Furthermore, Wu's method has an estimation failure problem in the range of practical mobile elevation angles. The objectives of this paper are two-fold: (1) to overcome these two problems with less arithmetic operation counts than Wu used; and (2) to improve the performance significantly. To achieve these objectives, we propose an antenna array configuration which avoids these problems. Simulation results verify that the proposed scheme can remove these problems and give much better performance.

Keywords: Array signal processing; Antenna array; Uniform linear array; Propagator method; Azimuth and elevation angles

1. Introduction

Many researchers have studied two-dimensional (2-D) direction of arrival angle (DOA) estimation for incident signals on an antenna element array [1–6]. It has also played an important role in many array signal processing fields such as radar, sonar, radio astronomy, seismic data processing, and mobile communication systems. MUSIC [7–9] and ESPRIT [10–12] have been regarded as the most
popular subspace algorithms that yield DOA estimations of high resolution. These algorithms have less computational complexity than the maximum likelihood methods in [13–14]. These algorithms, however, employ either eigen value decomposition (EVD) or singular value decomposition (SVD), which are still computationally extensive and time consuming especially when the number of antenna array elements \( N \) is larger than the number of incident signals. The EVD requires the computational complexity of order \( O(N^3 + MN^2L) \), where \( L, M, \) and \( N \) are the number of snapshots, the number of subarrays, and the number of antenna elements in each subarray respectively.

Many other efforts have been made to reduce the computational complexity of the EVD [15–16]; however, these techniques still require high computational complexity. Marcos proposed another DOA estimation scheme, called a propagator method (PM), without using any EVD or SVD [17] for one-dimensional (1-D) DOA estimations. The PM method can reduce the computational complexity to order \( O(MNLK) \) with insignificant signal-to-noise ratio (SNR) degradation [17–20] where \( K \) is the number of sources. The PM is a linear operator, which employs a least square (LS) criteria. Li et al. extended the PM in [18] to a 2-D DOA estimation problem [21], but the drawback of Li’s method is that it requires an exhaustive 2-D peak search through all possible steering vectors.

Recently, Wu et al. proposed a 2-D DOA estimation method using two parallel uniform linear arrays (ULAs), which were divided into three subarrays, \( X, Y, \) and \( Z \), as shown in Fig. 1(a) [22]. The Wu method does not require the 2-D peak search and therefore has a significantly lower computational complexity than the one used by Li [18]. Still, the Wu method has several drawbacks: (1) it requires a pair matching between the 2-D azimuth and elevation angle estimations; (2) it has an estimation failure problem when the elevation angles are between 70° and 90°; and (3) it has performance degradation at low SNRs, especially when the elevation angles are between 0° and 20° and the azimuth angles are close to 0° [22]. However, the elevation angles in typical mobile communication environments can be between 70° and 90°. Therefore, the Wu’s PM should be reconsidered for the mobile communication applications. This is the motivation of our paper.

The objectives of our paper are as follow: (1) to remove those problems in the Wu method [22]; and (2) to improve performance of the Wu method.
significant and achieve slightly smaller computational load than Wu used. To achieve these objectives, this paper proposes the antenna array configuration shown in Fig. 1(b) and also employs the PM in [18]. The significance of this paper involves its proposing a method: (1) that does not require any pair matching \((\phi_i, \theta_k)\) where \(\phi_i\) and \(\theta_k\) are the azimuth and elevation angle estimates for source \(i\) and source \(k\), respectively; (2) that has no elevation angle estimation failure even if the elevation angles are between 70° and 90°; and (3) that improves performance significantly, compared to Wu in [22].

Section 2 presents the proposed 2-D DOA method. Section 3 shows simulation results. And Section 4 makes conclusions.

2. Proposed antenna array configuration for 2-D DOA estimation

2.1. System model and analysis

Fig. 1(b) shows the proposed array configuration which consists of three uniform linear arrays with interspacing \(d\) equal to a half wavelength of incident signals where all sources use the same carrier frequency. The three uniform linear arrays in Fig. 1(b) consist of \(N\), \(N + 1\), and \(N\) elements respectively. One array is placed in the \(x-y\) plane, another on the \(y-z\) plane, and the last one in the \(y-z\) plane. Let \(X\), \(Y\), \(Z\), and \(W\) denote the 1st, 2nd, 3rd, and 4th subarrays of the proposed array configuration shown in Fig. 1(b). Each subarray consists of \(N\) elements. Suppose that there are \(K\) narrow band sources where the \(k\)th source has an elevation angle \(\theta_k\) and an azimuth angle \(\phi_k\), \(k = 1, \ldots, K\).

The \(N \times 1\) signal vectors received at the \(X\), \(Y\), \(Z\), and \(W\) subarrays at snapshot \(t\) are denoted, respectively, as

\[
X(t) = [x_1(t) \ x_2(t) \ \ldots \ x_N(t)]^T, \quad t = 1, 2, \ldots, T, \quad (1)
\]

\[
Y(t) = [y_1(t) \ y_2(t) \ \ldots \ y_{N+1}(t)]^T, \quad (2)
\]

\[
Z(t) = [z_1(t) \ z_2(t) \ \ldots \ z_N(t)]^T, \quad (3)
\]

\[
W(t) = [w_1(t) \ w_2(t) \ \ldots \ w_N(t)]^T, \quad (4)
\]

where superscript \(T\) represents the transpose, \(t\) the snapshot index, and \(L\) the number of snapshots. These \(N \times 1\) received vectors at snapshot \(t\) can be written as

\[
X(t) = A(\theta, \phi)S(t) + n_x(t), \quad (5)
\]

\[
Y(t) = A(\theta, \phi)\Phi_1(t)S(t) + n_y(t), \quad (6)
\]

\[
Z(t) = A(\theta, \phi)\Phi_2(t)S(t) + n_z(t), \quad (7)
\]

\[
W(t) = A(\theta, \phi)\Phi_3(t)S(t) + n_w(t), \quad (8)
\]

where

\[
A(\theta, \phi) = [a(\theta_1, \phi_1) a(\theta_2, \phi_2) \ldots a(\theta_K, \phi_K)], \quad (9)
\]

\[
a(\theta_k, \phi_k) = [1 \ u_k \ \ldots \ u_k^{N-1}]^T, \quad k = 1, \ldots, K, \quad (10)
\]

\[
u_k = \exp\left(-j\frac{2\pi d \sin \theta_k \sin \phi_k}{\lambda}\right), \quad (11)
\]

\[
S(t) = [s_1(t) \ s_2(t) \ldots s_K(t)]^T, \quad (12)
\]

for the \(K\) sources,

where \(\theta = [\theta_1, \ldots, \theta_K]^T\), \(\phi = [\phi_1, \ldots, \phi_K]^T\), \(S(t)\), is a signal vector at snapshot \(t\) for the \(K\) sources, and \(n_x\), \(n_y\), \(n_z\), and \(n_w\) are the \(N \times 1\) additive white Gaussian noise (AWGN) vectors whose elements have zero mean and variance \(\sigma^2\).

The matrices \(\Phi_1(\theta, \phi)\) in (6), \(\Phi_2(\theta)\) in (7), and \(\Phi_3(\theta, \phi)\) in (8) are \(K \times K\) diagonal matrices containing information about the elevation angle \(\theta_k\) and the azimuth angle \(\phi_k\) which can be written as

\[
\Phi_1(\theta, \phi) = \text{diag}[q_1 \ q_2 \ldots q_K], \quad (13)
\]

\[
q_k = \exp\left(-j\frac{2\pi d \sin \theta_k \sin \phi_k}{\lambda}\right), \quad (14)
\]

\[
\Phi_2(\theta) = \text{diag}[r_1 \ r_2 \ldots r_k], \quad (15)
\]

\[
r_k = \exp\left(-j\frac{2\pi d \cos \theta_k}{\lambda}\right), \quad (16)
\]

\[
\Phi_3(\theta, \phi) = \text{diag}[v_1 \ v_2 \ldots v_k], \quad (17)
\]

\[
v_k = \exp\left(-j\frac{2\pi d \sin \theta_k \cos \phi_k}{\lambda}\right), \quad (18)
\]

respectively, where \text{diag} means the diagonal elements of the matrix.
The matrices $\Phi_1(\theta, \phi), \Phi_2(\theta, \phi),$ and $\Phi_3(\theta, \phi)$ can be found by employing the PM [18] whose computational complexity is smaller than that of the subspace eigen analysis such as MUSIC and ESPRIT algorithms. We will drop the angle vectors $\theta, \phi$ and the snapshot index $t$ in the rest of the paper for the sake of simplicity. The PM in [18] makes a partition of the array response vector $A$ as follows:

$$A = [A_1^T \ A_2^T]^T, \quad (19)$$

where $A_1$ and $A_2$ are submatrices of dimension $K \times K$ and $(N - K) \times K$, respectively. Let us apply the $K \times (4N - K)$ propagator matrix $P$ to the observation output vectors $X, Y, Z,$ and $W$ in (5), (6), (7), (8), respectively. Further, let $D$ denote

$$D = [A^T \ A(\Phi_1)^T \ A(\Phi_2)^T \ A(\Phi_3)^T]^T \quad (20)$$

After partitioning $D$ in the similar way, it can be written as

$$D = [A_1^T \ D_1^T]^T, \quad (21)$$

where

$$D_1 = [A_2^T \ A(\Phi_1)^T \ A(\Phi_2)^T \ A(\Phi_3)^T]^T$$

Under the hypothesis that $A_1$ is a $K \times K$ non-singular matrix, which most subspace based methods assume when $N \geq 2K$, the $K \times (4N - K)$ propagator matrix $P$ is a unique linear operator which can be written as

$$P^HA_1 = D_1, \quad (23)$$

where the superscript $H$ is the Hermitian operator, i.e., the conjugate and transpose.

Let $Q(t)$ denote a $4N \times 1$ snapshot data vector written as

$$Q(t) = [X^T(t) \ Y^T(t) \ Z^T(t) \ W^T(t)]^T, \quad t = 1, \ldots, L. \quad (24)$$

Now let $F$ denote the $4N \times L$ data matrix consisting of $L$ snapshots as

$$F = [Q(1), Q(2), \ldots, Q(L),] \quad (25)$$

The $4N \times 4N$ cross spectral matrix, then, can be written as

$$\hat{R} = \frac{1}{L}FFH = \frac{1}{L} \sum_{i=1}^L Q(T)(T)^H. \quad (26)$$

The partitions of the data matrix $F$ and cross spectral matrix $\hat{R}$ can be written, respectively as

$$F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \quad (27)$$

and

$$\hat{R} = [E \ L], \quad (28)$$

where $F_1$ and $F_2$ are submatrices with dimension $K \times L$ and $(4N - K) \times L$, respectively, and $E$ and $J$ are sub-matrices with dimension $4N \times K$ and $4N \times (4N - K)$, respectively.

Let $\hat{P}_{\text{data}}$ and $\hat{P}_{\text{csm}}$ denote the $K \times (4N-K)$ and $K \times (4N-K)$ propagator estimate matrix based on the data matrix $F$ and cross spectral matrix $\hat{R}$, respectively. The propagator estimate matrix can be obtained by minimizing the following cost functions:

$$\zeta_{\text{data}}(\hat{P}_{\text{data}}) = ||F_2 - \hat{P}_{\text{data}}^HF_1||_F^2, \quad (29)$$

$$\zeta_{\text{csm}}(\hat{P}_{\text{csm}}) = ||J - E\hat{P}_{\text{csm}}||_F^2, \quad (30)$$

$$\hat{P}_{\text{data}} = (F_1F_1^H)^{-1}F_1F_2^H, \quad (31)$$

$$\hat{P}_{\text{csm}} = (E^HE)^{-1}E^HL. \quad (32)$$

We can partition either $\hat{P}_{\text{data}}$ or $\hat{P}_{\text{csm}}$ as follows:

$$\hat{P}_c = [\hat{P}_1^T \ \hat{P}_2^T \ \hat{P}_3^T \ \hat{P}_4^T \ \hat{P}_5^T \ \hat{P}_6^T \ \hat{P}_7^T]^T, \quad (33)$$

where the dimension of $\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{P}_4, \hat{P}_5, \hat{P}_6,$ and $\hat{P}_7$ are identical with the dimension of $A_2, A_1\Phi_1, A_2\Phi_1, A_1\Phi_2, A_2\Phi_2, A_1\Phi_3,$ and $A_2\Phi_3$, respectively.

According to (23), and from (22) and (33), we can write the following equations:

$$\hat{P}_1A_1 = A_2, \quad (34)$$

$$\hat{P}_2A_1 = A_1\Phi_1, \quad (35)$$

$$\hat{P}_3A_1 = A_2\Phi_1, \quad (36)$$
Using pair Eqs. (34), (36), (34), (38), and (34), (40), we can find $\Phi_1$, $\Phi_2$, and $\Phi_3$ by solving the following formulas, respectively:

$$
\hat{P}_3\hat{P}_1^\# A_2 = A_2\Phi_1,
$$

(41)

$$
\hat{P}_3\hat{P}_1^\# A_2 = A_2\Phi_2,
$$

(42)

$$
\hat{P}_3\hat{P}_1^\# A_2 = A_2\Phi_3,
$$

(43)

where $\#$ denotes the pseudoinverse.

This implies that the diagonal elements in the diagonal matrices $\Phi_1$, $\Phi_2$, and $\Phi_3$, respectively, can be estimated by finding the $K$ eigenvalues of each matrix $\hat{P}_3\hat{P}_1^\#$, $\hat{P}_3\hat{P}_1^\#$, and $\hat{P}_7\hat{P}_7^\#$ in (41)–(43). Moreover, $\Phi_1$, $\Phi_2$, and $\Phi_3$ can be found in another way, i.e., by finding the eigen-values of $\hat{P}_2$, $\hat{P}_4$, and $\hat{P}_6$ in (35), (37), and (39), respectively.

Using $\hat{P}_3\hat{P}_1^\#$, $\hat{P}_3\hat{P}_1^\#$, and $\hat{P}_7\hat{P}_7^\#$ yields more accurate results than using $\hat{P}_2$, $\hat{P}_4$, and $\hat{P}_6$, but the former requires computation loads larger than the latter. Then, the azimuth and elevation angle estimates for each source can be easily found as

$$
\hat{\phi}_k = \tan^{-1}\left[\frac{\text{arg}(\Phi)_k}{\text{arg}(\Phi_3)_k}\right],
$$

(44)

$$
\hat{\theta}_k = \tan^{-1}\left[\frac{\text{arg}(\Phi_3)_k}{\text{arg}(\Phi_2)_k \cos \hat{\phi}_k}\right],
$$

(45)

using $\Phi_1$, $\Phi_2$, and $\Phi_3$, in (13), (15), and (17).

2.2. Discussion

Note that the elevation angle estimate $\hat{\theta}_k$ can be found from (45) using the azimuth angle estimate $\hat{\phi}_k$ in (44). Both of them in (44) and (45) use the arc-tangent operator, which allows the value of its argument to be larger than the unity. One of the main advantages of the proposed method over the method in [22] is that the proposed method uses the arc-tangent operator for estimation of $\hat{\phi}_k$ and $\hat{\theta}_k$ from the different sources. These arc-tangent operations eliminate estimation failure. In other words, the arc-tangent operation $y = \tan^{-1}(x)$ is a one-to-one function for the range of $y \in (-\pi/2, \pi/2)$ and all $y \in \mathbb{R}$ where $\mathbb{R}$ denotes the real field domain. This implies that the arc-tangent values always exist for any $y \in \mathbb{R}$, and our proposed method never fails.

However, the method in [22, Eq. (22.14)] estimates the azimuth and elevation angles as follows:

$$
\hat{\phi}_k = \tan^{-1}\left[\frac{\text{arg}(\Phi_1)_k}{\text{arg}(\Phi_3)_k}\right],
$$

(46)

$$
\hat{\phi}_k = \tan^{-1}\left(\frac{\lambda}{2\pi d} \sqrt{(\text{arg}(\Phi_1)_k)^2 + (\text{arg}(\Phi_3)_k)^2}\right),
$$

(47)

where, for convenience, the notations in [22, Eq. (22.14)] were changed to match our notations. Note that Eq. (47) of the method in [22] uses arc-sine to estimate the elevation angle. This arc-sine function: $y = \sin^{-1}(x)$ is a one-to-one function only for $x \in [-1, 1]$ and $y \in (-\pi/2, \pi/2)$. In practice, the absolute value $|x|$ can be larger than one even if the SNR is high such as 10 dB. This situation most likely occurs when the elevation angles are between 70° and 90° which is a practical range in a mobile environment. Moreover, the failure rate increases to 50% when the elevation angle in [22] approaches to 90°. Refer to the explanations in detail [23].

Now, we compare the computational loads of the proposed method with those of the method in [22] using the same number of array elements in total, $N_{\text{total}}$. The number of multiplications in [22] is in the order of $O(3N'KL)$ to compute the propagator per trial where $N'$ is the number of elements in each subarray shown in Fig. 1(a), $N_{\text{total}} = 2N' + 1$, $L$ is the number of snapshots per trial, and $K$ is the number of sources. The proposed method uses an order of $O(4NLK)$ multiplications to compute the propagators in (31) or (32) per trial where $N$ is the number of elements in each subarray of the proposed array configuration shown in Fig. 1(b) where
For example, if the total number of elements is $N_{\text{total}} = 31$, then $N'$ will be 15 and the computational load for the method in [22] will be in order of $O(45LK)$ whereas they will be $N = 10$ and $O(40LK)$ for the proposed method, which implies that the proposed method has slightly less computational load than the method in [22]. This difference between the computational load of the two schemes increases as the number of array elements increases.

3. Simulation results

For simulation, the spacing between the two adjacent elements in any uniform linear array was set to a half wavelength of the incoming signals. Further, $L = 200$ number of snapshots per trial and 500 independent trials in total were tested. The root mean square error (RMSE) of the proposed DOA estimation scheme, using the array configuration in Fig. 1(a), was compared with that of the algorithm in [22] which employs the parallel-shape configuration in Fig. 1(b). The RMSE for the joint DOA estimation is defined as

$$\text{RMSE} = \sqrt{E[(\hat{\theta}_i - \theta_i)^2 + (\hat{\phi}_i - \phi_i)^2]},$$

(48)

where $i$ represents the source index, $E[X]$ denotes the expectation of a random variable $X$, and $(\hat{\theta}_i, \hat{\phi}_i)$ are the pair of the elevation and azimuth angle estimates.

Fig. 2 shows the RMSE in degrees of the azimuth and elevation angle estimates $(\hat{\theta}_i, \hat{\phi}_i)$ from $0^\circ$ to $90^\circ$ with $5^\circ$ increments for the parallel-shape algorithm [22]. We assumed $K = 1$ single source and $N_{\text{total}} = 15$ elements, and we set SNR = 10 dB. Fig. 3 shows the corresponding RMSE results for the proposed algorithm with $N_{\text{total}} = 13$ elements. We observe from Figs. 2 and 3 that the proposed algorithm improves the performance significantly compared to the parallel-shape array in [22]. The average of the RMSE value over all possible pairs $(\hat{\theta}_i, \hat{\phi}_i)$ for the proposed algorithm is 0.3923 in degrees whereas that for the parallel-shape algorithm in [22] is 0.9335 in degrees. In Fig. 2, only the successful trial cases were counted for the RMSE calculation.

We observe that the parallel-shape array in [22] shows many estimation failures when the elevation angles are between $70^\circ$ and $90^\circ$. As the elevation angle approaches $90^\circ$, the estimation failure rate of the parallel-shape array becomes 50%. Refer to (47) in this paper or [23, Section 3 Part C, pp. 2063 and Section 4 Part B, pp. 2064] to see the reasons why the parallel-shape algorithm in [22] fails when the elevation angle is between $70^\circ$ and $90^\circ$.

However, the proposed algorithm shows zero number of estimation failures for any pair of incident DOAs $(\theta_i, \phi_i)$ and all trials. Refer to (44)
and (45) to see the reasons why the PM algorithm with the proposed antenna array configuration in Fig. 2(b) shows no failures. Therefore, the PM method with the proposed antenna array configuration in Fig. 1(b) is more practical than the PM of the parallel-shape in [22] because typical elevation angles are between 70° and 90° in a mobile communications environment [24, pp. 19].

Tables 1 and 2 list the RMSE values and the number of estimation failures for both PMs with the parallel-shape in Fig. 1(a) [22] and the proposed shape in Fig. 1(b), respectively, with an elevation angle as a parameter from 72° to 90° with 3° increment for a given azimuth angle of 60°. A single source and three different SNR values −5, 0, and 5 dB are considered. We observe that the proposed configuration gives a better estimation than the parallel-shape array in [22] because the RMSE of the proposed method is approximately 2° smaller than that of the parallel-shape array. In addition, Table 2 confirms that the proposed antenna array configuration shows no estimation failure regardless of SNR values, but the RMSE increases as SNR decreases, whereas Table 1 indicates that the parallel shape array shows an unacceptable estimation failure rate as the SNR value becomes low or the elevation angle approaches 90°. Moreover, the estimation failure rate of the parallel shape array is almost 50% when the elevation angle approaches 90°.

Figs. 4 and 5 show the RMSE values of the joint elevation and azimuth DOA estimation versus the SNR in dB for source 1 and 2, respectively, when K = 2 source signals arrive with DOAs of (θ, φ) = (15°, 6°) and (30°, 50°). The total number of elements is $N_{total} = 13$ for both the proposed and parallel shape configurations. We observe that the proposed configuration is 5.4 dB and

<table>
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<tr>
<th>(φ, θ)</th>
<th>RMSE SNR = −5 dB</th>
<th>RMSE SNR = 0 dB</th>
<th>RMSE SNR = 5 dB</th>
<th># Failures SNR = −5 dB</th>
<th># Failures SNR = 0 dB</th>
<th># Failures SNR = 5 dB</th>
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<tr>
<td>(60°, 72°)</td>
<td>4.5025</td>
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Five hundred independent trials have been tested.

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<th>(φ, θ)</th>
<th>RMSE SNR = −5 dB</th>
<th>RMSE SNR = 0 dB</th>
<th>RMSE SNR = 5 dB</th>
<th># Failures SNR = −5 dB</th>
<th># Failures SNR = 0 dB</th>
<th># Failures SNR = 5 dB</th>
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<td>0.9321</td>
<td>0.4643</td>
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</table>

Five hundred independent trials have been tested.
2.5~3.3 dB better in SNR than the parallel shape in [22] for source 1 and source 2, respectively, for a given RMSE value, e.g., 0.5° or 1°.

4. Conclusions

An antenna array configuration was proposed for the 2-D azimuth and elevation angle estimation problem and compared with the parallel-shape configuration. The proposed method employs a PM which does not require any EVD or SVD but only a linear operation. The proposed 2-D DOA estimation scheme shows a significant improvement over the existing parallel shape scheme in [22]. In other words, (1) the proposed scheme does not require any pair matching for the 2-D DOA estimation problems whereas the parallel shape PM scheme in [22] does; and (2), the proposed algorithm shows no estimation failure for any pair of azimuth and elevation angles whereas the parallel-shape method in [22] can have a 50% failure rate when the elevation DOA approaches 90°.

References


