Iterative EM Based LDPC CDMA Receiver under Time Varying Interference

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Abstract — The authors of this paper proposed an iterative expectation-maximization (EM) channel estimation based on a low-density parity-check (LDPC) code-division multiple access receiver in their previous work. The receiver algorithm could efficiently estimate both channel coefficients and noise power spectral density (PSD). However, their previous work considered only a single user channel environment, so they tested only for a constant noise PSD. In the current paper, we extend the single user to a more realistic multiuser environment for the testing of a time-varying interference plus noise PSD estimation. Also our proposed adaptive PSD estimation scheme will be compared with a conventional system in which a constant PSD is assumed. It will show that the proposed iterative EM scheme can provide accurate interference plus noise PSD estimates to the LDPC decoder.

Index Terms— EM Algorithm, Iterative receiver, LDPC

I. INTRODUCTION

In a realistic multiuser scenario the desired user data is normally corrupted by other undesired users at random time intervals, depending on other users transmission activities. In this case if a low-density parity-check (LDPC) decoder uses a constant value of noise power spectral density (PSD) estimation, e.g., $N_0$, then its performance will be degraded. Conventional receivers are such examples because they attempt to ascertain signal-to-noise ratio (SNR) by measuring received signal strength at the receiver front end and assuming a known constant noise PSD. The authors of this paper proposed an iterative expectation-maximization (EM) channel estimation based LDPC code-division multiple access (CDMA) receiver in [1]. The receiver algorithm could efficiently estimate both channel coefficients and noise PSD. However, a single user channel environment was considered, so in [1] only a constant noise PSD was tested.

In this current paper, we extend the single user to a more realistic multiuser environment. In other words, the proposed algorithm in [1] will be tested under a time-varying interference plus noise PSD environment. This is because in practice, interference plus noise PSD or partial-band jamming PSD are typically time varying. Also our proposed adaptive PSD estimation scheme will be compared with a conventional system in which a constant PSD is assumed, under a realistic time varying PSD model.

Typically a conventional system employs a complicated interference cancellation or interference suppression scheme in a multiuser environment. For the comparisons, it is assumed that the conventional system does not employ such interference cancellation or suppression scheme because the proposed system does not use such schemes. Even if our proposed system does not have any interference cancellation scheme, it will show reasonable performance because the proposed iterative EM scheme can provide more accurate interference plus noise PSD estimates to the LDPC decoder than the conventional system.

Section II describes the system model including the channel. Section III presents the proposed EM-based iterative LDPC receiver including code-division multiple-access (CDMA) spreading. Section IV shows simulations results, and finally Section V draws conclusions.

II. SYSTEM MODEL

Fig. 1 shows a block diagram of multuser transmitters where each transmitter consists of an LDPC encoder, spreading, and binary phase-shift keying (BPSK) modulation. User (A) is the desired user and User (B) is the interference. Input to an encoder in Fig. 1 is a binary independent identically distributed data source from a user. A block of length $m$, i.e., $\mathbf{v}^{(a)} = [v_1^{(a)}, \ldots, v_m^{(a)}]$ with $v_i^{(a)} \in [1,0]$, is taken by the User (A)’s encoder. Similarly, a block of length $m$, $\mathbf{v}^{(b)} = [v_1^{(b)}, \ldots, v_m^{(b)}]$ with $v_i^{(b)} \in [1,0]$, is taken by the User (B)’s encoder. Assume that there are only two states for the users’ transmission activities, i.e., “ON” and “OFF.” User (A) is always in “ON” state whereas User (B) changes its state periodically with a duty cycle of 10%.

A. Encoding and Modulation

The same encoding and modulation schemes used in [1] are employed for both users. Refer to [1] for the detail notations and explanations. The encoded sequence vectors $\mathbf{u}^{(A)}$ and $\mathbf{u}^{(B)}$ are spread by User (A)’s and User (B)’s spreading codes, respectively. Also, Users (A) and (B) are assumed to be chip synchronized for the convenience. These spread codes are not perfectly orthogonal. Hence, the interfering signal coming from User (B) would not be completely suppressed by the User (A)’s desprader.

B. Channel Model

User (A)’s and User (B)’s fading coefficients are generated independently. Also, fading blocks of a user
are independent. The same complex block fading model used in [1] is employed for both users, where fading amplitudes and phases are constant over a fading block. For example, each frame of 2000 code symbols is broken into 50 subframes and a subframe consists of 40 code symbols and 4 pilot symbols. Each subframe is multiplied by a spreading block of 44 symbols.

\[
\begin{align*}
\mathbf{v}^{(i)} &\rightarrow \mathbf{u}^{(i)} &\rightarrow \text{Spreading} &\rightarrow \text{MUX} &\rightarrow \text{BPSK} &\rightarrow \text{User (A) Signal} \\
\mathbf{v}^{(j)} &\rightarrow \mathbf{u}^{(j)} &\rightarrow \text{Spreading} &\rightarrow \text{MUX} &\rightarrow \text{BPSK} &\rightarrow \text{User (B) Signal}
\end{align*}
\]

Fig. 1. Two user transmitters for multiuser simulation.

C. Iterative Receiver Structure

Again, the same receiver structure used in [1] is employed by User (A). Fig. 2 shows the block diagram for User (A)'s receiver.

\[
\begin{align*}
\text{Chip Matched Filter} &\rightarrow \mathbf{y}_k &\rightarrow \text{Spreading Sequence (SS)} &\rightarrow \text{Despreader} &\rightarrow \text{LDPC Decoder} \\
&\rightarrow \text{Channel Estimator} &\rightarrow \text{Pilot Symbols} &\rightarrow \text{MUX} &\rightarrow \text{BPSK} &\rightarrow \text{User (A) Signal} \\
&\rightarrow \text{Pilot Symbols} &\rightarrow \text{MUX} &\rightarrow \text{BPSK} &\rightarrow \text{User (B) Signal}
\end{align*}
\]

Fig. 2. The iterative receiver for User (A).

The complex envelope of the output, which is sampled every chip, can be written as

\[
y_k = C_{(\beta)}^{(A)} x_k^{(A)} + C_{(\beta)}^{(B)} x_k^{(B)} + n_k, \quad 1 \leq k \leq n_g, \tag{1}
\]

for the first code word, where the superscripts (A) and (B) denote Users (A) and (B), respectively.

\[
y_k = \text{received data at the } k^{th} \text{ chip in a code word},
\]

\[
C_{(\beta)}^{(A)} = \left[ C_{[k/(n_g g)]}^{(A)} \right] e^{j \left[ \text{kn} g \right]} \quad \text{and}
\]

\[
C_{(\beta)}^{(B)} = \left[ C_{[k/(n_g g)]}^{(B)} \right] e^{j \left[ \text{kn} g \right]} \quad \text{are independent complex block fading coefficients for User (A) and (B), respectively, whose magnitudes and phases are constant over the } \left[ k/(n_g g) \right]\text{-th fading block.}
\]

\[
\left[ \gamma \right] \text{ is the largest integer less than or equal to } \gamma,
\]

\[
x_k^{(A)} = p_k^{(A)} u_k^{(A)} \quad \text{and} \quad x_k^{(B)} = p_k^{(B)} u_k^{(B)} \quad \text{are code symbols for User (A) and (B), respectively at the } k^{th} \text{ chip index},
\]

\[
\beta = \left( (k-1)/g \right) + 1, \quad \beta = 1, \ldots, n,
\]

\[
p_k = \text{spreading code at the } k^{th} \text{ chip},
\]

\[
n_k = \text{a complex AWGN sample at the } k^{th} \text{ chip with}
\]

\[
F \left[ p_k \right] = N_0,
\]

\[
g = \text{number of chips per code symbol, and}
\]

\[
n = \text{the number of code symbols in a code word.}
\]

Refer to [1] for notations in detail.

A receiver iteration is also defined as in [1]. For example, \( j_{\text{max}} = 10 \) receiver iterations, and for each \( j \), \( i_{\text{max}} = 20 \) decoder iterations followed by \( i_{\text{max}} = 10 \) EM internal iterations.

Let \( \hat{\theta}_{(j)}^{(i)} = (\hat{C}_{(j)}^{(i)}, \hat{j}_{(j)}^{(i)}) \) represent the estimates of the channel coefficient and interference plus noise PSD level parameters at the \( j^{th} \) internal iteration for the \( i^{th} \) closed-loop receiver. Refer to [1] for the detailed explanation and operation of Fig. 2.

III. ITERATIVE LDPC CDMA RECEIVER

A. EM Algorithm

Theoretically, ML estimation \( \hat{\theta} \) can be obtained using a received data vector \( y = [y_1, y_2, \ldots, y_{n_g}] \) referred to as incomplete data, by maximizing the conditional log-likelihood function as \( \hat{\theta} = \arg \max \ln f(y | \theta) \). However, the computation of this equation is almost prohibitive in practice. Hence, the expectation of the conditional log-likelihood of \( z = (y, x) \), known as complete data, i.e., \( E_{\hat{\theta} | z}[\ln f(x | \hat{\theta})] \), is iteratively maximized with respect to \( \theta \), where expectation is taken with respect to \( x \) given \( y \) and \( \theta \). This is why the algorithm is called an expectation maximization (EM) algorithm. On considering the complete data and using the derivations in [1], this can be written as

\[
\hat{\theta} = \arg \max_\theta E_{\hat{\theta} | z}[\ln f(y | x, \hat{\theta})]. \tag{2}
\]

This leads to our objective function

\[
\chi(\theta | \theta^{(i)}) = -\ln f_{\theta^{(i)}} [\sum_{k = 1}^{n_g} |y_k|^2 + C_{(\beta)}^{(i)}] + \frac{2}{L_0} \sum_{k = 1}^{n_g} |\text{Re}(y_k p_k C_{(\beta)}^{(i)})| \tag{3}
\]

where

\[
L_0 = \begin{cases} N_0 & \text{if User (B) is OFF} \\ N_0 + E_{\beta} T_s/T_c & \text{if User (B) is ON} \end{cases} \tag{4}
\]

\( T_c = \text{chip interval, and } T_s = \text{code symbol interval.} \)

From the objective function, we obtain the channel coefficient estimate for User (A) as

\[
\hat{C}_{(i)}^{(j)} = \frac{1}{n} \sum_{k = 1}^{n_g} y_k p_k \hat{C}_{(\beta)}^{(j)} \tag{5}
\]

where

\[
\hat{C}_{(i)}^{(j)} = \text{User (A)’s fading channel coefficient estimate at the } (i+1)^{th} \text{ EM and } j^{th} \text{ receiver iteration, and}
\]

\[
\hat{C}_{(\beta)}^{(j)} = \text{expectation of User (A)’s } \beta^{th} \text{ code symbol at the } i^{th} \text{ EM and } j^{th} \text{ receiver iteration.}
\]

User (B)’s fading channel coefficients are not necessary to be estimated because interference
cancellation is not required in the proposed system. Also, from the objective function in (3), the estimate of the interference plus noise PSD is obtained as
\[
\hat{I}_{(i)}^{(j)}(t) = \frac{1}{ng} \sum_{k=1}^{\infty} \left[ |\hat{c}^{(i)}_{(j)}|^{2} + 2 \sum_{k=1}^{\infty} \text{Re}(y_{k}^{*}p_{k}^{(j)} \hat{c}^{(i)}_{(j)} \hat{p}_{(j)}) \right].
\] (6)

Again, it is not necessary to separately estimate \( N_{0} \) from \( I_{0} \) because the \( I_{0} \) estimation in (6) includes both the effects of the AWGN and the interference PSD on the system as explained in (4).

The channel estimate \( \hat{\theta}_{(j)} = (\hat{C}^{(i)}_{(j)}, \hat{I}^{(j)}_{(j)}) \) is fed into the LDPC decoder for the \( j \)-th receiver iteration. Then, the LDPC decoder with this channel estimate decodes the demodulated code symbols iteratively. The probability values of \( s_{j}(\beta) = \Pr[u_{\beta} = -1|y] \) are stored for each code symbol \( \beta \) at the final LDPC decoding iteration, and are fed back into the EM channel estimator, \( \beta = 1, \ldots, n \).

Using this feedback, the expectation of \( u_{\beta} \) at the \( i \)-th EM and \( j \)-th receiver iteration can be found as
\[
\pi^{(i)}_{\beta}(j) = \frac{1 - s_{j}(\beta) - s_{j}(\beta)}{1 - s_{j}(\beta) + s_{j}(\beta)} \frac{f(y_{\beta} | u_{\beta} = -1, \hat{\theta}^{(i)}_{(j)})}{f(y_{\beta} | u_{\beta} = +1, \hat{\theta}^{(i)}_{(j)})}.
\] (7)

Let us define parameter \( R^{(i)}_{\beta}(j) \) as [2]
\[
R^{(i)}_{\beta}(j) = \frac{f(y_{\beta} | u_{\beta} = -1, \hat{\theta}^{(i)}_{(j)})}{f(y_{\beta} | u_{\beta} = +1, \hat{\theta}^{(i)}_{(j)})} = \exp \left[ \sum_{k=\beta} \frac{4p_{k} \text{Re}(y_{k}^{*} \hat{C}^{(i)}_{(j)})}{\hat{I}^{(j)}_{(j)}} \right].
\] (8)

IV. SIMULATION RESULTS

The LDPC decoding algorithm in [3] was used for our simulations. Figure 3 shows BER versus \( E_{b}/N_{0} \) at receiver iterations \( j = 0, 1, 3, 5, 7 \) and 9 with adaptive estimation of fading coefficient and the interference plus noise PSD level. Block fading coefficients for User (A) and User (B) were generated by an independent Rayleigh block fading model. User (A) is always “ON” whereas User (B) is turned “ON” and “OFF” periodically. The duty cycle of User (B) is 10\% i.e., in every 10 frame, the 9 frames are of User (A) affected by just noise, whereas in the 10\textsuperscript{th} frame User (B) data causes interference. The performance improvement between receiver iteration 0 and 1 is about 1.3 dB at 10\textsuperscript{-3} BER. The gain between 1\textsuperscript{st} and 9\textsuperscript{th} receiver iteration is 1 dB.

Fig. 4 shows BER versus \( E_{b}/N_{0} \) under the same environment as of Fig. 3 for \( j = 0, 1, 3, 5, 7 \) and 9 receiver iterations with only estimating fading coefficients whereas keeping interference plus noise PSD \( I_{0} \) constant. The performance improvement between non-adaptive \( I_{0} \) PSD scheme and adaptive \( I_{0} \) estimation scheme for \( j = 9 \) is significant such as 6.5 dB at 10\textsuperscript{-3} BER.

Obviously, performance of the adaptive scheme in Fig. 3 is better than that of the non-adaptive scheme in Fig. 4. This is because when User (B) is “ON” it causes interference to User (A) data, and User (A) receiver can estimate this \( I_{0} \) level change correctly. On calculating this increased interference level and passing it to LDPC decoder aids in decoding better than feeding constant \( I_{0} \) PSD. In addition, from Figs. 3 and 4, it is observed that the non-adaptive PSD case shows a BER floor even for a high SNR such as 25 dB whereas the proposed adaptive \( I_{0} \) PSD scheme shows 0 BER beyond the last points for all receiver iterations.

Fig. 3: BER versus \( E_{b}/N_{0} \) for block length \( n = 2000 \) code bits and receiver iterations \( j = 0, 1, 3, 5, 7 \) and 9 with adaptive \( I_{0} \) and fading coefficient estimation for independent Rayleigh block fading.

Fig. 4: BER versus \( E_{b}/N_{0} \) for block length \( n = 2000 \) code bits and receiver iterations \( j = 0, 1, 3, 5, 7 \) and 9 with non-adaptive \( I_{0} \) and fading coefficient estimation for independent Rayleigh block fading.

V. CONCLUSIONS

This paper tested a refined iterative receiver proposed in [1] under time-varying interference plus noise PSD level \( I_{0} \), assuming two active users. One of them was always “ON” for the desired user model and the other was “ON” and “OFF” periodically with a fixed duty cycle. The proposed adaptive PSD estimation scheme with fading coefficient estimation shows a significant gain over a non-adaptive scheme of constant PSD equal to \( N_{0} \) which has been typically assumed in conventional receivers. Therefore, the proposed receiver can be applied showing better performance than the conventional
schemes under a time-varying interference plus noise PSD environment.

REFERENCES

