Combined Tone and Noise Jamming Against Coded FH/MFSK ECCM Radios

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Abstract— It is known that partial-band tone jamming (PBTJ) is generally the worst form of jamming for frequency-hopped M-ary FSK (FH/MFSK) communication systems. Recent studies show that for some coded systems, full-band noise jamming (FBNJ) is more effective than worst-case PBTJ if a receiver is able to utilize jamming state information (JSI) for decoding, when the symbol energy-to-uniform noise jamming power spectral density ratio \( \frac{E_s}{N_j} \) is small. In this paper, we conjecture that a proper combination of PBTJ and FBNJ under a given total jamming power constraint may be more effective than PBTJ alone, not only for the case with low \( E_s/N_j \) but also for the case with high \( E_s/N_j \), since the FBNJ can corrupt the JSI. Assuming this combination of PBTJ and FBNJ jamming, we consider three cases of receiver processing—the hard decision (HD) metric without JSI, the HD metric with perfect JSI, and the maximum likelihood (ML) metric using Viterbi’s ratio threshold (VRT) to generate a 1-bit decision quality indicator. System performance is evaluated in terms of the Chernoff bound on the probability of symbol error. From extensive numerical analysis we conclude the following. For the case of the HD metric without JSI, PBTJ-only jamming is the worst form of jamming as expected since the receiver does not use JSI at all; for the other cases, a combination of PBTJ and FBNJ is the worst, with the worst ratio of PBTJ power to FBNJ power a function of the values of \( M \) and \( E_s/N_j \).

I. INTRODUCTION

In an electronic warfare environment, where a battle is waged between the communicating party and a jammer who is intent on disrupting the communicator’s link, strategy plays an important and fundamental role for the opposing parties. In this paper, we analyze and evaluate the effectiveness of sophisticated jamming waveforms in degrading the performance of a frequency-hopped M-ary frequency shift keying (FH/MFSK) communications system which utilizes various ECCM (electronic countermeasures) techniques to mitigate the jamming effects. The FH/MFSK system considered in this paper is a scheme which can provide the communicator with jam-resistant radio capabilities. The most successful type of jamming against FH/MFSK radios has been shown to consist of placing equal power jamming tones such that, at most, one of the \( M \)-ary signaling frequencies is jammed on a given hop, called “\( n = 1 \) band multitone jamming” [1, vol. II, p. 413], [7]. In this paper, we call this jamming simply “tone jamming” or “partial-band tone jamming” (PBTJ).

Recent studies show that for some coded systems, full-band noise jamming (FBNJ) is more effective than worst-case PBTJ if a receiver is able to utilize jamming state information (JSI) for decoding, especially when the symbol energy-to-uniform noise jamming power spectral density ratio \( \frac{E_s}{N_j} \) is small [1, vol. II, p. 178], [7]. In this paper, we conjecture that a proper combination of PBTJ and FBNJ under a given total jamming power constraint may be worse than PBTJ alone, not only for the case with low \( E_s/N_j \) but also for the case with high \( E_s/N_j \), since the FBNJ can corrupt the JSI.

Because the tone jamming power is concentrated at certain frequencies, a JSI generator in the FH/MFSK receiver can easily detect whether a jamming signal is present or not during a symbol transmission. Such JSI can be exploited by the ECCM receiver’s decoder to emphasize the un jammed symbol and to deemphasize the jammed symbol [1–3]. As the jamming power becomes stronger and stronger, the JSI about the PBTJ is more and more reliable. Sometimes, a strong PBTJ may actually improve the communication link performance if perfect JSI is available. Hence, a more intelligent jammer will not use PBTJ only against an ECCM receiver with a JSI generator. Instead, the jammer may choose to combine PBTJ with FBNJ because the ECCM radio cannot easily detect FBNJ, and therefore FBNJ can cause the ECCM radio receiver to generate inaccurate JSI about the PBTJ. It is the objective of this paper to investigate how well a coded FH/MFSK system with JSI that has been fortified against PBTJ alone withstands a simultaneous onslaught of both FBNJ and PBTJ. We assume the total jamming power is fixed and the portion of total jamming power used in FBNJ can be controlled to maximize the jamming effects.

Assuming this combination of PBTJ and FBNJ jamming, we consider three cases of receiver processing—the hard decision (HD) metric without JSI, the HD metric with perfect JSI, and the maximum likelihood (ML) metric using Viterbi’s ratio threshold (VRT) to generate a 1-bit decision quality indicator [4], [5], [8]. We consider a VRT receiver in this paper because it is a simple and practical antitone-jamming receiver. In addition,

This is identical to the effect of classical additive Gaussian noise (AWGN), except that the channel corruption is caused by a broadband jammer.
the VRT techniques can be regarded as imperfect side information\footnote{Actually, this side information is not restricted to JSI, that is, the quality bit indicates the general quality of the channel itself.} generators. Therefore, the ML metric with VRT is a good candidate for the purpose of comparison to the HD metric for the cases of perfect JSI and no JSI. System performance is evaluated in terms of the Chernoff bound on the probability of symbol error \cite{1, eq. 2.28}.

Many who are familiar with coding will appreciate the value of the Chernoff bound's $D$ parameter in the computation of the error probability bounds for given code structures and in the comparison of cutoff rates, which are practically achievable code rates for the memoryless coding channel \cite{1-5}. Here the meaning of coding channel is the effective channel as seen by the encoder and decoder system (see Fig. 1). Once the $D$ parameter is obtained, the cutoff rate and bit error probability bound can be calculated for given specific codes \cite{1, ch. 4}. Hence, we will evaluate $D$ parameters for coded FH/MFSK systems. First, we will maximize $D$ parameters with respect to the fraction of hopping slots jammed by the PBTJ when a portion of the total jamming power is assigned to FBNJ and the rest of total jamming power is used by PBTJ. Second, the worst-case combined jamming with respect to the portion of total jamming power used in FBNJ against the three cases of receivers mentioned above will be numerically calculated for some specific $E_b/N_j$. Finally, we plot $D$ versus $E_b/N_j$ with the portion of total jamming power used in FBNJ as a parameter.

The paper is organized as follows. A general system description is given in Section II. In Section III, the HD metric cases are analyzed. Section IV examines the ML metric case with imperfect JSI generated by the VRT. In Section V, numerical results are discussed, and conclusions are given in Section VI.

II. SYSTEM DESCRIPTION

Our analysis neglects thermal noise, which is assumed to be dominated by the effects of the jammer. We assume contiguous MFSK modulation and one hop per transmit-

![Fig. 1. A coded frequency-hopped $M$-ary frequency shift keying communication system overview.](image)

ted symbol. In addition, we consider "$n = 1$ partial-band multitone jamming" \cite[vol. II, p. 79]{} which jams at most one of the $M$-ary signaling frequencies. The PBTJ chooses randomly the jamming portion of the total frequency-hopped system bandwidth $W$.

Let $J$ be the total jamming power and $\epsilon$ be a portion of $J$ used by the FBNJ, $0 \leq \epsilon \leq 1$. Then the PBTJ uses power $(1 - \epsilon)J$. Let $N_j = J/W$ denote the uniform noise jamming power spectral density over the total frequency-hopped system bandwidth $W$, with $N = \epsilon N_j/T_h$ as the FBNJ power measured in a frequency hopping slot, whose bandwidth is the inverse of a hopping time interval $T_h$. Let $N_{fh} = WT_h$ denote the total number of frequency hopping slots, $q$ the number of jamming tones chosen by the PBTJ, $I$ the power in a single tone (equal to $(1 - \epsilon)J/q$), $S$ the received signal power, $E_s$ the symbol energy, and $K = \log_2 M$ the number of bits in a channel symbol. Then, the fraction of hopping symbols jammed by the PBTJ, $\rho$, is given by

$$\rho = \frac{Mq}{N_{fh}}, \quad 0 < \rho \leq 1. \tag{1}$$

This $\rho$ corresponds to the probability that any symbol in a contiguous $M$-ary band is tone-jammed. (Our $\rho$ is $\mu$ in \cite[vol. II, eq. (2.28), p. 80]{1}.) Note that this $\rho$ is also analogous to the fraction of the full spread-spectrum bandwidth $W$ jammed in the case of partial-band noise jamming. The worst-case PBTJ chooses $\rho$ to make the FH/MFSK system have the worst performance for given PBTJ power, $(1 - \epsilon)J$. The signal-to-FBNJ power ratio $S/N$, the single jamming tone-to-FBNJ power ratio $I/N$, and the signal-to-single tone power ratio $S/I$ are then

$$S/N = \frac{1}{N - \epsilon N_j}, \quad I/N = \frac{(1 - \epsilon)M}{\rho \epsilon}, \quad S/I = \frac{\rho E_s/N_j}{(1 - \epsilon)M}. \tag{2}$$

The ratios in (2) will be frequently used in the analysis and numerical computations of $D$ parameters in Sections III-V for a given $M$ and $E_s/N_j$. 
Let the vector \( z_i \) denote an \( M \)-dimensional PBTJ state vector whose \((M - 1)\) components are 0 for frequency slots unjammed, and 1 for the \( i \)th frequency slot jammed, for an \( M \)-ary channel orthogonal symbol transmission, \( i = 1, \cdots, M \). Let \( z_0 \) mean that no \( M \)-ary symbols are tone jammed. Since at most one of the \( M \)-ary signaling frequencies is tone jammed, there are \((M + 1)\) possible PBTJ states. The probabilities of possible PBTJ states are then

\[
Pr[z_i] = Pr[z_1] = \cdots = Pr[z_M] = \frac{\rho}{M},
\]

\[
Pr[z_0] = 1 - \rho. \tag{3}
\]

Without loss of generality, we can assume the transmitted channel symbol is the first symbol because the \( M \)-ary symbols are equiprobable.

Under a PBTJ state vector \( z_i \), the transmitted signal and the jamming tone fall in the same frequency slot, and the conditional probability density functions of envelope detector outputs, \( R_1, \cdots, R_M \), after dehopping [1, vol. I, p. 206], are

\[
p(R_i | z_i) = R_1 N \left[ \frac{A_i R_i}{N} \right] \exp \left( \frac{-R_i^2 + A_i^2}{2N} \right), \tag{4a}
\]

for the signal channel, and

\[
p(R_i | z_0) = \frac{R_1}{N} \exp \left( \frac{-R_i^2}{2N} \right), \quad k = 2, \cdots, M. \tag{4b}
\]

for the nonsignal channels, where \( I_0 \) is the modified Bessel function of the first kind, \( A_i^2 = 2S + 2I + 2\sqrt{2S} \sqrt{2I} \cos \phi \), and \( \phi \) is the relative tone phase to signal, uniformly distributed over \([0, 2\pi]\).

Under a PBTJ state vector \( z_i \), the signal and the tone fall in different frequency slots, and the conditional probability densities [1, vol. I, p. 206] are

\[
p(R_i | z_i) = R_1 N I_0 \left( \frac{2SR_i}{N} \right) \exp \left( \frac{-R_i^2 + 2S}{2N} \right), \tag{5a}
\]

for the unjammed signal channel, and

\[
p(R_i | z_0) = R_i N I_0 \left( \frac{\sqrt{2I} R_i}{N} \right) \exp \left( \frac{-R_i^2 + 2I}{2N} \right), \tag{5b}
\]

for the jammed nonsignal channel, and

\[
p(R_i | z_0) = \frac{R_i}{N} \exp \left( \frac{-R_i^2}{2N} \right), \quad k = 3, \cdots, M \tag{5c}
\]

for the unjammed nonsignal channels.

Under \( z_0 \), no channels are tone jammed, and the probability density of \( R_i \) is that shown in (5a), and the probability densities for nonsignal channels are given in (4b).

For a memoryless \( M \)-ary orthogonal coding channel, and given jammer’s choices of \( \epsilon \) and \( \rho \), the cutoff rate \( R_0 \) is a function of the parameter \( D \) [1, vol. I, pp. 193–195], given by

\[
R_0(\epsilon, \rho) = 1 - \log_M \left[ 1 + (M - 1)D(\epsilon, \rho) \right], \tag{6}
\]

where

\[
0 \leq D(\epsilon, \rho) = \min_{\lambda} D(\epsilon, \rho, \lambda) \leq 1. \tag{7}
\]

where \( \lambda \) is the Chernoff probability bound parameter and

\[
D(\epsilon, \rho, \lambda) = \mathbb{E} \left[ \left( \exp \left( \lambda \left[ \mu(Y, X = m', Z) - \mu(Y, X = m, Z) \right] \right) \right) | X = m, m' \neq m \right]. \tag{8}
\]

In (8), “\( \mathbb{E} \)” means the expectation over the random received vector \( Y \) (here \( Y = R \)) and PBTJ state vector \( Z \). The metric \( \mu(y, x = m, z) \) is the receiver’s decision metric when the transmitted symbol is \( m, 1 \leq m \leq M \), and the PBTJ state vector is \( z \). We assume \( m = 1 \) for the transmitted symbol, and \( m' = 2 \) for the nontransmitted symbol, without loss of generality. The \( D \) parameter is the Chernoff bound on the probability that the decision metric for the nontransmitted symbol, \( m' = 2 \), is larger than that for the transmitted symbol, \( m = 1 \), on a single symbol transmission. The average of conditional \( D \) parameters over possible tone jamming conditions is then

\[
D(\epsilon, \rho, \lambda) = Pr(z_i) D(\epsilon, \rho, \lambda | z_i)
+ \Pr(z_0) D(\epsilon, \rho, \lambda | z_0)
+ (M - 2) \Pr(z_0) D(\epsilon, \rho, \lambda | z_i)
+ \Pr(z_0) D(\epsilon, \rho, \lambda | z_0). \tag{9}
\]

Jammer’s goal is to maximize \( D(\epsilon, \rho) \) by choosing the best \( \epsilon \) and \( \rho \). The general relation of the parameter \( D \) to the coded bit error probability is \( P_b \leq G(D) \), where \( G(\cdot) \) is a function determined solely by the specific code, whereas the parameter \( D \) depends only on the coding channel and the decoder metric [1, vol. I, pp. 194, 199].
III. HARD DECISION METRICS

When hard decisions are made and JSI about PBTJ is available, the metric is

\[
\mu(y, x = m, z_i) = \begin{cases} 
  c_i & \text{if } y_m > y_{m'} \text{ for all } m' \neq m \\
  0 & \text{otherwise},
\end{cases}
\]

for an \( M \)-dimensional observation vector \( y \) of envelopes where the weighting coefficients are

\[
c_i = \begin{cases} 
  1 & \text{for } i = 0, 1, \\
  c & \text{for } i = 2, \ldots, M.
\end{cases}
\]  

A practically implementable JSI generator with weights (10a) can be built as follows [1, vol. II, last paragraph, p. 113]. When only one energy detector output among \( M \) detector outputs is high on a given transmission, use relatively large weight 1, and when two or more energy detector outputs are high, use relatively small weight \( c \). In this weighting, the decoder makes use of the PBTJ event that helps the communication link, and discards the PBTJ events that disturb the communications, together with the transmitted information. If there is only PBTJ, then such JSI becomes perfect. But if FBNJ shares the total jamming power with PBTJ, then such JSI cannot be perfect because FBNJ can cause the JSI generator to generate wrong JSI about PBTJ. In this paper we assume that this JSI with weights (10a) is perfect for an ideal case. Using the HD metric with perfect JSI in (10) and (10a), we can express the parameter \( D \) in (9) as follows:

\[
D(\rho, \lambda, c) = \frac{\rho}{M} \left[ e^{-\lambda} (1 - (M - 1)A) + e^{\lambda} A + (M - 2)A \right] + \frac{(M - 1)\rho}{M} \left[ e^{-\lambda} B + e^{\lambda} 1 - B \right] + \frac{(M - 2)(1 - B)}{M - 1} + (1 - \rho) \left[ e^{-\lambda} (1 - (M - 1)C) + e^{\lambda} C + (M - 2)C \right]
\]

where

\[
A = \Pr[R_{m'} > R_j \text{ for all } j \neq m', m' \neq 1|z_i]
\]

\[
= \sum_{k=0}^{M-2} \binom{M-2}{k} (-1)^k \frac{1}{(1+k)(2+k)} \cdot \exp \left[ \frac{S + I 1 + k}{N 2 + k} \right] I_0 \left( \frac{2\sqrt{SI} 1 + k}{N 2 + k} \right).
\]  

As an extreme case, assume that there is no FBNJ and only PBTJ is active, i.e., \( \epsilon = 0 \), and that the perfect JSI
is available. From (11), probabilities $A$ and $C$ are zero and probability $B$ is
\[
B = \begin{cases} 
1 & \text{if } S/N \geq 1 \\ 
0 & \text{otherwise}. 
\end{cases} \tag{17}
\]
Then, the parameter $D$ in (12) can be simplified to the well-known form \[1, \text{ vol. II, eq. (2.138)}\]
\[
D = \begin{cases} 
\frac{M - 1}{E_r/N_j} & \text{if } E_r/N_j \geq M \\ 
\frac{M - 1}{M} & \text{otherwise}. 
\end{cases} \tag{18}
\]
As another extreme case, suppose only FBNJ is active, i.e., $E = 1$. From (11), probabilities $A$ and $B$ are equal to zero, and the parameter $D$ in (12) becomes
\[
D = \begin{cases} 
2\sqrt{\alpha \beta} + \gamma & \text{if } \alpha \geq \beta \\ 
1 & \text{otherwise}; \tag{19}
\end{cases}
\]
where
\[
\alpha = (1 - (M - 1)C), \quad \beta = C, \quad \gamma = (M - 2)C. \tag{20}
\]
In (19) and (20), the probability $C$ is as in (11c) with $\epsilon = 1$.

B. Hard Decision Metric Receiver without JSI
Suppose the receiver cannot derive JSI. Then the metric must be independent of the PBTJ states, which implies $c = 1$ in (11). After minimizing $D(\epsilon, \rho, \lambda)$ with respect to $\lambda$, we have
\[
D(\epsilon, \rho) = \begin{cases} 
2\sqrt{\alpha \beta} + \gamma & \text{if } \alpha \geq \beta \\ 
1 & \text{otherwise}, \tag{21}
\end{cases}
\]
where
\[
\alpha = \frac{\rho}{M} \left(1 - (M - 1)A\right) + \frac{\rho}{M} (M - 1)B + (1 - \rho)(1 - (M - 1)C), \\
\beta = \frac{\rho}{M} A + \frac{\rho}{M} (1 - B) + (1 - \rho)C, \\
\gamma = \frac{\rho}{M} (M - 2)A + \frac{\rho}{M} (M - 2)(1 - B) + (1 - \rho)(M - 2)C. \tag{22}
\]
When $\epsilon = 0$ (tone jamming) and JSI is not available, $A = C = 0$ in (11) and the parameter $D$ in (21) becomes
\[
D = \max_{0 < \rho < \min(1, E_r/N_j)} \left[2\sqrt{\left(\frac{\rho}{M} + 1 - \rho\right)\frac{\rho}{M}} + (M - 2)\frac{\rho}{M}\right]. \tag{23}
\]
If $M = 2$, then (23) becomes well-known result \[2\]
\[
D = \begin{cases} 
1 & \text{if } E_r/N_j < 2 \\ 
2\sqrt{\left(1 - 1/(E_r/N_j)\right)/(E_r/N_j)} & \text{otherwise}. \tag{24}
\end{cases}
\]
When the FBNJ-only is active against FH/MFSK with the HD metric without JSI, the $D$ parameter is the same as that for the HD metric with perfect JSI in (19), since JSI about PBTJ is meaningless.

IV. ML METRIC WITH VITERBI'S RATIO THRESHOLD TECHNIQUE
For the purpose of comparison to the HD cases discussed in Section III, we consider the VRT receiver \[4, \tag{5}, \tag{8}\] which can be regarded as a receiver with imperfect side information about the channel (not just PBTJ state information). In \[5\], the performance of the VRT receiver against PBTJ in the presence of background noise, which is not under the control of jammer, was investigated. In this section, we take the same model as in \[5\], with the following three main differences. First, here the FBNJ power can be controlled by the jamming in contrast to \[5\]. We will numerically try to find the worst-case value of $\epsilon$, the portion of total jamming power used in FBNJ. Second, in \[5\], a Gaussian quadrature numerical integration method was used for computation of the parameter $D$, while in this paper we avoid numerical integrations as much as possible by expanding the corresponding probability expression in binomial expansion form, and by using Massaro's results \[6, \tag{eq. (16)}\]. Third, our analysis is given for the original $M$-ary channel, which is in contrast to the binary decomposed channel analyzed in \[5\].

A coding channel with VRT techniques takes $M$-ary input symbols interleaved (e.g., 1, 2, 3, or 4 in Fig. 3 for $M = 4$) and produces $2M$-ary soft decision symbols. The $2M$-ary soft decision symbol consists of a hard decision symbol (from MFSK maximum envelope detector) and a quality bit $Q$ (from VRT side information generator) (e.g., $1G, 1B, \ldots, 4B$ in Fig. 3 for $M = 4$). After deinterleaving, the quality bit is exploited to predict which bits are jammed, and hence discount potentially bad decisions in the decoder. The quality bit $Q$ is derived as follows. If the ratio of the largest of the filter output envelopes, $R_1, \ldots, R_M$, to the next largest is bigger than a threshold $\theta$, then $Q = G$ (good), otherwise $Q = B$ (bad). The threshold value is one of the parameters that can be controlled by the receiver. Notice that there are many parameters, i.e., $\epsilon, \rho,$ and $\theta$, for our worst-case analysis. To make the problem solvable in a reasonable time period, we fix the threshold value, $\theta$, in the numerical analysis of the next section.

The coding channel with $M$-ary inputs and $2M$-ary outputs can be characterized by discrete transition probabilities, $P_C$ (correct, $Q = G$), $P_{EX}$ (correct, $Q = B$), $P_{EX}$ (error, $Q = B$), and $P_{ex}$ (error, $Q = G$). In Fig. 3, a coding channel with 4-ary input and 8-ary output is shown as an illustration similar to the analysis in \[5\]. These tran-
From (5) and (6), the conditional correct exceeding probabilities (31)-(33) given the jamming states can be derived as

$$\bar{F}_c(\theta, \rho | z_1) = \Pr \bigg[ \bigcap_{j=2}^{M} (R_j \geq \theta R_j) \bigg| z_1 \bigg]$$

$$= \sum_{k=0}^{M-1} (-1)^k \left( \frac{M-1}{k} \right) \frac{\theta^2}{\theta^2 + k} \cdot \exp \left[ -\frac{k}{k + \theta^2} \cdot \frac{S + I}{N} \right] \times I_0 \left[ \frac{2\sqrt{k}}{N} \frac{k}{k + \theta^2} \right].$$

$$\bar{F}_c(\theta, \rho | z_2) = \Pr \bigg[ \bigcap_{j=2}^{M} (R_j \geq \theta R_j) \bigg| z_2 \bigg]$$

$$= \sum_{k=0}^{M-2} (-1)^k \left( \frac{M-2}{k} \right) \frac{\theta^2}{\theta^2 + k} \cdot \exp \left[ -\frac{k}{k + \theta^2} \cdot \frac{S}{N} \right] \times \left\{ 1 - Q(\sqrt{2a}, \sqrt{2b}) + \frac{1}{1 + \theta^2 + k} \right\} \cdot \exp \left[ -(a + b)I_0(2\sqrt{ab}) \right].$$

and

$$\bar{F}_c(\theta, \rho | z_0) = \Pr \bigg[ \bigcap_{j=2}^{M} (R_j \geq \theta R_j) \bigg| z_0 \bigg] = (31)$$

with \( I = 0 \),

$$(33)$$

where \( a = (I/N)(\theta^2 + k)/(1 + \theta^2 + k) \) and \( b = (S/N)\theta^2/((\theta^2 + k)(1 + \theta^2 + k)) \). Note that if \( \theta = 1 \), then (32) becomes (11b) for ordinary HD metric. Similarly,
we can derive a binomial expansion form for $\tilde{F}_r$ as

$$\tilde{F}_r(\theta, \rho) = \frac{\rho}{M} \times (35) + (M - 1) \frac{\rho}{M} \times (37)$$

$$+ (1 - \rho) \times (36),$$

$$\tilde{F}_r(\theta, \rho | z_i) = \Pr \left\{ \bigcap_{j=1}^{M} (R_j \geq \theta R_j), \ j \neq 2 | z_i \right\}$$

$$= \sum_{k=0}^{M-2} \binom{M-2}{k} (-1)^k \frac{\theta^2}{\theta^2 + k} \times \exp \left[ \frac{-S + I}{N} \frac{\theta^2 + k}{\theta^2 + k} \right]$$

$$\times I_0 \left\{ \frac{2 \sqrt{SI}}{N} \frac{\theta^2 + k}{\theta^2 + k} \right\}$$

$$= \sum_{l=0}^{M-1} \binom{M-1}{l} \cdot (-1)^l \frac{\theta^2}{\theta^2 + l - 1} \frac{1}{\theta^2 + l - 1}$$

$$\times \exp \left[ -S + I \frac{\theta^2 + l - 1}{\theta^2 + l} \right]$$

$$\times I_0 \left\{ \frac{2 \sqrt{SI}}{N} \frac{\theta^2 + l - 1}{\theta^2 + l} \right\}$$

$$= \frac{1}{M - 1} \times (38) + (M - 2) \frac{1}{M - 1} \times (39),$$

where

$$\Pr \left\{ \bigcap_{j=1}^{M} (R_j \geq \theta R_j), \ j \neq 2 | z_i \right\}$$

$$= (32) \text{ replaced } S \text{ by } I, \text{ and } I \text{ by } S,$$

$$\Pr \left\{ \bigcap_{j=1}^{M} (R_j \geq \theta R_j), \ j \neq 3 | z_2 \right\}$$

$$= \sum_{k=0}^{M-3} \binom{M-3}{k} (-1)^k \frac{\theta^2}{\theta^2 + k} \times \exp \left[ \frac{-S}{N} \frac{\theta^2 + k}{\theta^2 + k} \right]$$

$$+ \exp \left[ - \frac{I}{N} \frac{\theta^2 + k}{\theta^2 + k} \right] - 1$$

$$+ \int_0^\infty e^{-r^2/2} \left\{ \sqrt{2S/N} \cdot \frac{t}{\sqrt{\theta^2 + k}} \right\}$$

$$\cdot \left\{ \sqrt{2I/N} \cdot \frac{t}{\sqrt{\theta^2 + k}} \right\} dt.$$  (39)
\( \epsilon = 0 \) extreme case), if perfect JSI is available to the decoder. The reason for this result is that PBTJ-only can cause the receiver to have a maximum \( D \) equal to \((M - 1)/M\) [see (18)] while the FBNJ-only can cause \( D \) to be equal to 1 if \( E_s/N_0 \) is small. Finally, as \( \epsilon \) changes from zero to one, the combined jamming becomes the more effective jamming, because the FBNJ becomes more effective with increasing FBNJ power, while the PBTJ effect does not change much with decreasing PBTJ power.

We observe the following three facts in Fig. 4(d) for a high \( E_s/N_0 \) example. First, \( D(E_s/N_0, \epsilon, \rho) \) increases and then decreases as \( \rho \) varies from zero to one for a given \( E_s/N_0 \) and \( \epsilon \). The peaks occur at \( \rho \) less than \( M/(E_s/N_0) \) \[ = 0.20047 \text{ in Fig. 4(d)} \] at which \( D \) is maximum for the PBTJ-only. [See (2), (17), and (18).] Second, the PBTJ-only is the more effective jamming than the FBNJ-only, which is a known result [1, Vol. II, p. 178], if perfect JSI is available to the decoder. The reason for this is that the \( D \) for the PBTJ-only with the worst-case jamming fraction \( \rho \) is a linearly inverse function of \( E_s/N_0 \) [see (18)] while \( D \) for the FBNJ-only is an exponentially decreasing function of \( E_s/N_0 \) [see (19)]. and, after the crossing point of the two curves, the exponentially decreasing function drops faster than the linearly inverse one. Finally, we observe in Fig. 4(d) that \( D(E_s/N_0, \epsilon, \rho^*) \), for some combined jamming (for example, \( \epsilon = 0.1 \)) with the worst jamming fraction, denoted by \( \rho^* \), can be larger than that of the PBTJ-only, if \( E_s/N_0 \) is high.

For intermediate \( E_s/N_0 \) examples, \( D(E_s/N_0, \epsilon, \rho) \) versus \( \rho \) with \( \epsilon \) as a parameter is shown for a given \( E_s/N_0 \) in Fig. 4(b) and (c). We can see that there is a tradeoff between FBNJ power and PBTJ power in order to make the HD metric with perfect JSI have the worst performance.
In Fig. 5, we draw $D(E_s/N_J, \epsilon, \rho^*)$ versus $\epsilon$ with $E_s/N_J$ as a parameter to see the worst case $\epsilon$ for $M = 4$. $D(E_s/N_J, \epsilon, \rho^*)$ versus $E_s/N_J$ with $\epsilon$ as a parameter is shown in Fig. 6. The corresponding results for $M = 2$ are shown in Figs. 7 and 8. The FBNJ-only is the worst-case jamming if $E_s/N_J$ is less than 1.14 dB for $M = 4$ (see Fig. 6), and the FBNJ-only is the worst case jamming if $E_s/N_J$ is less than 8.23 dB for $M = 2$ (see Fig. 8). For other $E_s/N_J$, as shown in Figs. 6 and 8, we can design more effective combined jamming than the PBTJ-only or the FBNJ-only, to be active against the HD metric with perfect JSI.

**B. Hard Decision Metric Receiver without JSI**

For the HD metric without JSI, $D(E_s/N_J, \epsilon, \rho)$ is maximized by some worst-case value of the jamming fraction, denoted $\rho^*$, for given $\epsilon$ and $E_s/N_J$. The maximized value of $D$, $D(E_s/N_J, \epsilon, \rho^*)$, is inversely proportional to both $\epsilon$ and $E_s/N_J$, as shown in Fig. 9. Furthermore, as $\epsilon$ increases from 0 to 1, the combined jamming scheme uniformly becomes less effective, which says that PBTJ alone ($\epsilon = 0$) is the most efficient jamming, and FBNJ alone is ($\epsilon = 1$) the least, when the JSI is not available to the HD metric.

**C. Maximum Likelihood Metric Receiver with VRT**

The receiver with VRT techniques can change the threshold value to minimize the combined jamming effect. In this paper, however, we fix the threshold value equal to 2.5 for our numerical analysis because this threshold value was shown to be near optimum for the performance of evaluation of ratio threshold for FH/MFSK under PBTJ plus background noise [5]. In Fig. 10, for an FH/4FSK system using an ML metric with VRT,
Fig. 9. $D$ versus $E_s/N_0$ for 4-ary FSK, hard decision without tone jamming state information and $\epsilon$ as a parameter (step 0.1).

Fig. 10. $D$ versus $\rho$ for 4-ary FSK, maximum likelihood metric receiver with Viterbi's ratio threshold techniques of threshold 2.5 and $\epsilon$ as a parameter (step 0.1) when $E_s/N_0 = 3$ dB (a), 7 dB (b), 10 dB (c), 13 dB (d).
curves of $D(E_s/N_t, \epsilon, \rho)$ versus $\rho$ are shown with $\epsilon$ as a parameter, for four different values of $E_s/N_t$.

For the extreme case $\epsilon = 0$ (the PBTJ-only case), we observe that there are two disconnected functions if $\rho_1 = M/(\theta E_s/N_t) \leq 1$, i.e., $E_s/N_t \geq M/\theta^2$. The first function is $D_1(\rho)$ in (41) for $0 \leq \rho \leq \rho_1$, and the second one is $D_2(\rho)$ for $\rho_1 < \rho \leq \rho_2$ [see (41)]. These two pieces are monotonically increasing with $\rho$, and local maxima occur at the ends of each piece. If $\rho_1 > 1$, i.e., $E_s/N_t < M/\theta^2$ (low $E_s/N_t$), then for the PBTJ-only with the worst-case $\rho$, $D$ is equal to 1 [see (41)].

When the FBNJ begins to share total jamming power with the PBTJ (i.e., $\epsilon > 0$), the behavior of the $D(E_s/N_t, \epsilon, \rho)$ versus $\rho$ curves in Fig. 10 becomes smooth, although we can still observe two local peaks for small $\epsilon$. As $\epsilon$ approaches 1, we generally observe one peak.

$D(E_s/N_t, \epsilon, \rho^*)$ is plotted against $\epsilon$ with $E_s/N_t$ as a parameter in Fig. 11, and versus $E_s/N_t$ with $\epsilon$ as a parameter in Fig. 12, for 4FSK with the ML metric. In Fig. 12, we observe that PBTJ-only is the most efficient jamming, with $D = 1$, if $E_s/N_t \leq M/\theta^2 = -1.93$ dB (i.e., low $E_s/N_t$), for $M = 4$ and $\theta = 2.5$ [see (41)]. We observe also that, for high $E_s/N_t$ (for example, $E_s/N_t \geq 9.43$ dB in Fig. 12), the PBTJ alone ($\epsilon = 0$) is again the most efficient jamming against the ML metric with VRT. We explain this result as follows. Assume that only PBTJ is active at first. Now, let the FBNJ take a little jamming power of the total jamming power, the PBTJ loses that amount of jamming power. Then, the FBNJ contribution to the jammer is less sensitive to the type of receiver, while the PBTJ effect due to the reducing of PBTJ power is more sensitive if $E_s/N_t$ is high. We can observe this sensitivity depending on the receiver type of comparing Fig. 12 to Figs. 6 and 9. From the comparison of the gap between the $\epsilon = 0$ and $\epsilon = 1$ extreme cases for high $E_s/N_t$ in Figs. 6, 9, and 12, the HD metric without JSI is seen to be the most sensitive to an incremental PBTJ power change, the ML metric with VRT less sensitive, and the HD metric with perfect JSI is the least sensitive. Also, for high $E_s/N_t$, the ML metric with VRT behaves as the HD metric without JSI with reduced signal power. (Compare the first equation of (41) with (23) for $\epsilon = 0$ extreme case. They are equivalent.) This is why the PBTJ alone is the most efficient jamming against the ML metric with VRT for high $E_s/N_t$.

In the medium range of $E_s/N_t$, the behavior of the ML metric with VRT is similar to that of the HD metric with perfect JSI (see Figs. 6 and 12). The most efficient jamming involves a combination of tone and noise.

Notice that the $D$ parameter is constant, 0.5, for the interval of 5 dB to 6 dB, when only the PBTJ is active (see Fig. 12). The reason for this result is that the maximum of two peaks in the middle equation of (41) is $D_2(1) = (M - 2)/M$ (equal to 0.5 if $M = 4$) for this interval.

The corresponding results for $M = 2$ are shown in Figs. 13 and 14. The general behavior of the $M = 2$ case is similar to that of $M = 4$.

Many days of CPU time were used to obtain one $D(E_s/N_t, \epsilon, \rho^*)$ versus $E_s/N_t$ curve for a given $\epsilon$ in the
ML metric with VRT receiver when a VAX-780 machine was used with a time-sharing load of 2-3. We could not produce sufficient data for many $\epsilon$'s because of the long computation. With our data, it is, however, interesting to try to draw the upper envelopes of $D(E_s/N_R, \epsilon, \rho^*)$ versus $E_s/N_R$ curves for the comparison of three receivers. They are shown in Figs. 15 and 16.

In Fig. 15 for BFSK, we observe that the ML metric with VRT is the best among three receivers for the medium range of $E_s/N_R$, $-4.57 \text{ dB} < E_s/N_R < 5.73 \text{ dB}$, and the HD metric with perfect JSI is the best for $E_s/N_R \leq -4.75 \text{ dB}$ or $E_s/N_R \geq 5.73 \text{ dB}$. The difference between these two receivers is less than 1 dB in the $E_s/N_R$ for a given $D$ if $E_s/N_R$ is less than 5.73 dB, and the difference increases for $E_s/N_R \geq 5.73 \text{ dB}$. The HD metric without JSI is the worst among the three receivers for any $E_s/N_R$, and 4 to 7 dB (or more) worse in $E_s/N_R$ for a given $D$ than the receiver with the second worst performance.

In Fig. 16, for 4-ary FSK, we observe that for the medium range of $E_s/N_R$, $5 \text{ dB} < E_s/N_R < 10.86 \text{ dB}$, the ML metric with VRT is the best, the HD metric with perfect JSI is the second best, and the HD metric without JSI is the worst. For $E_s/N_R \leq 5 \text{ dB}$ or $E_s/N_R \geq 10.86 \text{ dB}$, the HD metric with perfect JSI is the best, the ML metric with VRT is second, the HD metric without JSI is the worst. The difference in $E_s/N_R$ between the HD metric with perfect JSI and the ML metric with VRT is less than 2.3 dB for low or high $E_s/N_R$, and less than 1 dB for the medium range of $E_s/N_R$. The HD metric without JSI is $3.42 \text{ dB}$ to $6.29 \text{ dB}$ worse for a given $D$ than the receiver with the second worst performance.

VI. CONCLUSIONS

We have considered combined tone and noise jamming against coded FH/MFSK systems under given total jamming power. The performances of three receivers were examined when combined jamming was used against them and evaluated in terms of the $D$ parameter.

From the numerical analysis, we observe the following. As expected for the HD metric without JSI, PBTJ alone is the worst-case jamming from the communicator's view, as expected, since the receiver does not use JSI at all. For the HD metric with perfect JSI, if $E_s/N_R$ is low, then FBNJ-only is the worst-case and PBTJ-only the least efficient jamming; otherwise, the most efficient portion of the total jamming power used in FBNJ is determined by the values of $M$ and $E_s/N_R$. For the ML metric with VRT, if $E_s/N_R$ is high, or low as $E_s/N_R \leq M/\theta^2$, then PBTJ-only is the worst-case; otherwise, the most efficient portion of the total jamming power used in FBNJ depends on $M$ and $E_s/N_R$. If the three receivers are compared to each other when the worst-case combined jamming is used against them, then the HD metric without JSI is the worst receiver for any $E_s/N_R$, the HD metric with perfect JSI is the best receiver for high or low $E_s/N_R$, and the ML metric with VRT is the best receiver for the medium range of $E_s/N_R$.
The difference in $E_r/N_0$ between the case of the HD metric with perfect JSI and the case of the ML metric with VRT is less than 1 dB for the medium range of $E_r/N_0$ to achieve the same $D$. As even more powerful hybrid jamming scenario is the combination of PBTJ and two-level partial-band noise jamming in place of the less effective FBNJ. However, the corresponding analysis is much more difficult, and this major extension of the authors' current paper is reserved for a follow-up effort, which will also include $M \geq 8$-ary FSK results.

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