Limiter-Differential Detection of a Frequency-Hopped CPFSK Diversity System in Partial-Band Jamming

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Abstract—The error probability achieved by a differential detector with a bandpass limiter preceding the receiver is analyzed for a slow-frequency-hopped CPFSK diversity waveform transmitted over a partial-band noise jamming channel, and is compared to the system's performance without the bandpass limiter. The system's thermal noise is not neglected in the analysis. In principle, each bit is repeated on \( L \) different hops and for the FH/CPFSK system analyzed, these repetitions are combined to yield a soft decision. The main result of the paper is that a diversity gain for error rate improvement in worst-case partial-band jamming is realized for the differential detector preceded by a limiter, but not for the differential detector without a limiter. This important result is shown by considering the error probability for \( L = 2 \) in comparison with that for \( L = 1 \).

I. INTRODUCTION

CONTINUOUS-phase frequency-shift-keying (CPFSK) modulations have received much attention over recent years because of their bandwidth efficiency [1]-[5]. CPFSK signals are continuous in phase, and the symbols are not generally orthogonal unless the FM modulation index \( h \) is a multiple of \( \frac{1}{2} \), unlike orthogonal FSK schemes [2, p. 52]. A coherent CPFSK demodulator employs the continuity of the phase to improve performance. In most applications, noncoherent receivers using either differential or limiter–discriminator detection are used because they are simpler and hence less expensive than coherent receivers. Unfortunately, they are often very difficult to analyze because of their nonlinear nature, which requires consideration of non-Gaussian noise in the performance analysis.

CPFSK is the basic waveform employed, for example, in the design of frequency-hopping combat net radios, such as SINCGARS, in the 30-90 MHz band. Frequency-hopped CPFSK (FH/CPFSK) is known to provide as much as 4 dB advantage over hopped M-ary FSK (FH/MFSK) when limiter–discriminator detection is employed [6]. Simon and Wang [7] presented performance analyses of coded FH/CPFSK with limiter–discriminator detection and hard-decision diversity combining; their results were based on an upper bound for the symbol error probability, and thermal noise was neglected. Later, Simon and Wang [8] compared the performance of differential detection with limiter–discriminator detection for CPFSK modulations. Miller et al. [9], [10] performed detailed analysis of uncoded FH/CPFSK for both limiter–discriminator detection and differential detection with soft-decision diversity combining; they concluded that a linear (unweighted) diversity combining of receiver samples does not produce a diversity gain for error rate improvement. Recently, it has been shown that weighting the limiter–discriminator or differential detector samples in inverse proportion to the total noise power on each hop does produce a gain [11].

The purpose of this paper is to present error probability analysis of the soft-decision limiter-differential detection receiver for two hops/bit FH/CPFSK diversity waveforms under worst-case partial-band jamming conditions, and to compare the results with those reported in [10] for differential detection without a limiter. Limiter-differential detection was found not to have been analyzed in the literature as a receiver to mitigate jamming effects for FH/CPFSK systems in partial-band noise jamming, and the motive for the analysis presented in this paper was the following observation: Since the output of the differential detector is proportional to the incoming total envelope as well as to the sine of the differential phase, it seemed likely that normalization of the receiver output samples to remove the envelope (and therefore any potential domination of the diversity sum by the sample from a jammed hop) would produce a diversity gain. Such normalization prior to the diversity sum did bring about improvement for FH/MFSK [12], [13]. The use of a bandpass limiter in front of a differential detector has the effect of normalizing the envelope, and thus is a form of nonlinear diversity combining. For soft-decision diversity combining, investigation of two-fold diversity is sufficient to determine whether a gain will be realized.

II. SYSTEM MODEL AND PARAMETERS

Our studies concern the jammed performance of hopped binary FM communications, particularly under the assumption of partial-band noise jamming and the use of time diversity to mitigate the effects of jamming. Fig. 1 gives a block diagram of the transmission scheme for the system. Binary data, \( d(t) \), are to be transmitted using slow-frequency-hopped digital FM, or CPFSK. The binary data are to be repeated on \( L \) different hops in order to increase...
the likelihood that some of the symbols are free of jamming. The figure suggests one of many possible ways to accomplish this objective. According to the version shown in the figure, the binary data symbols are first read into a Q-bit shift register (Q-symbol buffer), where Q is the number of symbols that can be transmitted in one hop period.

When the Q symbols have all been generated at rate $R_b$ and stored in the buffer, they are then transferred to a second (output) buffer. The transmitter logic then reads this buffer $L$ times at the rate $LR_b$, and this stream of data "chips," $c(t)$, is used to frequency-modulate the selected frequency-hopping carrier frequency, which is changed (hopped) to a new pseudorandomly-selected value after Q chips have been transmitted. In this manner, $L$ copies of the Q-symbol sequence have been transmitted on $L$ different successive hops. The Q symbols in this paper are assumed to be repeated in the same order, although it is possible to scramble them to achieve a different order on each hop. The original data rate $R_b$ equals the chip rate $R$, divided by $L$, on account of the repetitions. Viewed another way, the energy transmitted per chip is the fraction $1/L$ of the data bit energy. The hop rate is $L/Q$ times the data bit rate because $Q$ data bits are transmitted over $L$ hop intervals. For a slow-frequency-hopping scheme, we have $Q > L$, so that a hop interval is larger than a bit interval.

The receiver, diagrammed in Fig. 2, must accurately synthesize local oscillator frequencies in order to dehop the signal at the proper times, and must then be capable of sampling the demodulator output at the ends of each chip interval. These samples ($QL$ of them for $L$ complete hops) are buffered so that the receiver logic can, in some specified manner, combine the $L$ chips belonging to a particular data bit. The output of the diversity combining is used to make the final bit decision. Although it is possible that some kind of transient effect will make the likelihood of bit errors dependent upon the position of the chip in the hop, $q$, or upon the number of chips per hop, $Q$, such effects are not considered in this paper. We assume that the average bit error probability is uniform with respect to these parameters.

The signal $r(t)$ at the input to the IF filter during the dwell time of a particular hop is:

$$r(t) = \sqrt{2S} \cos (2\pi f_0 t + \theta(t)) + w(t) \tag{1}$$

where $S$ is the signal power and $f_0$ is the IF filter center frequency. It is assumed that a jammer, with power $J = N_j W$, chooses to concentrate his power in the fraction $\gamma$ of the total hopping bandwidth, $W$, so that the noise term $w(t)$ is considered to be white Gaussian noise with one-sided spectral density $N_0$ if the hop is not jammed, or $N_0 + N_j/\gamma$ if the hop is jammed by a partial-band noise jammer. It is assumed that $\gamma$ is chosen by the jammer to maximize the communicator's average error probability. In (1), $\theta(t)$ is the chip phase waveform, and is given by:

$$\theta(t) = \frac{\pi h}{T} \int_{-\infty}^{t} c(\tau) d\tau + \theta_0 \tag{2}$$

in which $h = 2f_c T = 2f_c/R$, is the modulation index and $\theta_0$ is a random initial carrier phase. Since noncoherent differential detection is employed by the system studied, the
value of \( \theta_0 \) does not affect the analysis, and this parameter can be neglected.

The IF filter is chosen to be Gaussian-shaped, with the transfer function

\[
H(f) = e^{-6f^2/2B^2}
\]

where \( B \) is the two-sided noise bandwidth of the IF filter. The output of the IF filter \( H(f) \) can be written as:

\[
x_{IF}(t) = 2\sqrt{2}\sigma(t) \cos(2\pi f_0 t + \phi(t)) + n(t)
\]

where \( \sigma(t) \) and \( \phi(t) \) are the time-varying amplitude and distorted phase of the filtered signal, and \( n(t) \) is narrowband Gaussian noise. The output \( y(t) \) of the bandpass limiter can be expressed as:

\[
y(t) = \cos(2\pi f_0 t + \phi(t) + \eta(t))
\]

where \( \eta(t) \) is the phase noise, which can be written as:

\[
\eta(t) = \tan^{-1} \left( \frac{\xi(t)}{\sqrt{2\rho(t)} + \xi(t)} \right)
\]

in which \( \xi(t) \) and \( \eta(t) \) are independent Gaussian variables with zero means and unit variances, and \( \rho(t) \) is the time-varying signal-to-noise ratio given by:

\[
\rho(t) = \begin{cases} 
\frac{E_b}{LN_0} \frac{a^2(t)}{BT} & \text{hop not jammed, with probability } 1 - \gamma \\
\frac{E_b}{LN_0 + N_s/\gamma} \frac{a^2(t)}{BT} & \text{hop jammed, with probability } \gamma
\end{cases}
\]

and \( E_b = ST_s \) is the signal bit energy.

For the limiter-differential detection receiver shown in Fig. 2, the sample \( z_{qi} \) from the \( q \)th chip on the \( i \)th hop is:

\[
z_{qi} = \frac{1}{2} \sin \Psi_{qi}
\]

where \( \Psi_{qi} \) is the sample phase change over a chip time interval as in limiter-discriminator detection. The samples are summed to produce a single decision statistic for the \( q \)th bit

\[
z_q = \sum_{i=1}^{L} z_{qi}
\]

so that the final bit decision \( d \) is made according to the rule:

\[
d = \begin{cases} 
1, & z_q \geq 0 \\
0, & z_q < 0
\end{cases}
\]

Pawula [4] has suggested that, for most cases of practical interest, the system bandwidth-time product \( BT \) is in the range of \( 1 \leq BT \leq 3 \) and the frequency deviation ratio \( h \) is less than 1.5, so that it is sufficient to consider intersymbol interference (ISI) effects due to immediately adjacent bits. He showed, for narrowband digital FM with limiter-discriminator detection, that the average bit error probability over three-bit (or three-chip, in this case) ISI patterns is almost the same as an average over longer patterns, as obtained by Tjhung and Wittke [3]. In addition, Simon and Wang [8] used this three-bit data pattern average for analyzing the bit error probability of a receiver with differential detection, where without diversity the decision statistic is given by \( \frac{1}{2} \sigma(t) a(t - T) \cos \Psi \), \( \Psi \) being the sample phase change over a bit interval. In this paper, the effects of ISI will also be analyzed by averaging with respect to three-chip ISI patterns.

The average bit error probability can be written as an average over three-chip ISI patterns, \( c = (111), (010), \) and \( (011) \) ([4], [7]–[10]):
The conditional error probabilities shown in (11) are discussed in the following section.

III. EXACT PERFORMANCE OF FH/CPFSK FOR L = 1, 2 WITH LIMITER-DIFFERENTIAL DETECTION

In this section, we find the exact error probabilities of \( L = 1 \) and 2 hops/bit FH/CPFSK with limiter-differential detection. These results are sufficient for establishing whether a diversity gain is realized by the combination of a bandpass limiter and the differential detector. The probability density function of the sampled differential phase \( \Psi_q \) is given in integral form in [4] and [5]. The final bit decision statistic \( z_q \) is an \( L \)-fold summation of the nonlinear functions, \( z_q = \frac{1}{2} \sin \Psi_q \).

\( L = 1 \) hop/bit: Because \( \sin \Psi_q = \sin (\Psi = \Psi_q \mod 2\pi) \), the performance can be evaluated by using the modulo-\( 2\pi \) probability density function of \( \Psi \) for \( \Psi - \Delta \phi \leq \pi \), where \( \Delta \phi \) is the value of the differential phase for the given intersymbol interference pattern when there is no noise or jamming. The conditional error probability for \( L = 1 \) can be written as follows [8]:

\[
\Pr(E|c, l \text{ hops jammed}) = \Pr(\sin \Psi < 0|c, l \text{ hops jammed}) = 1 - \Pr(0 \leq \Psi \leq \pi|c, l \text{ hops jammed}) = F(0|c, l) - F(\pi|c, l), \quad l = 0, 1;
\]

where \( F(\alpha) \) is related to the cumulative distribution function of \( \Psi \) and is given in [5].

\( L = 2 \) hops/bit: Since \( \sin \Psi_q = \sin (\Psi_q \mod 2\pi) \), we restrict \( \Psi_q \) to be in the interval \( \Delta \phi \pm \pi, i = 1, 2 \). We delete the index \( q \) in what follows, assuming that the error probability does not depend on the position of the chip in the \( Q \)-chip hop transmissions. The decision regions in the plane for ‘‘1” and ‘‘0” chip values \( I_1 \) and \( I_2 \), respectively, are:

\[
I_1 = \{(\Psi_1, \Psi_2): \sin \Psi_1 + \sin \Psi_2 \geq 0|c, l \text{ hops jammed}\}
\]

\[
I_2 = \{(\Psi_1, \Psi_2): \sin \Psi_1 + \sin \Psi_2 < 0|c, l \text{ hops jammed}\}
\]

and \( \Delta \phi - \pi \leq \Psi_1, \Psi_2 \leq \Delta \phi + \pi \).

Fig. 3 gives an example of \( I_2 \) for the signal phase change, \( \Delta \phi = \frac{\pi}{2} \). (In general \( \Delta \phi \) depends on the chip pattern.) The conditional error probability is the integral of the joint probability density function of \( \Psi_1 \) and \( \Psi_2 \) over the shaded regions; independence of differential phase samples is assumed, so that \( p_{\Psi_1, \Psi_2}(x, y) = p_{\Psi_1}(x)p_{\Psi_2}(y) \).

Note that, for any \( \Delta \phi > 0 \), the lines \( AB (\Psi_1 + \Psi_2 = 0) \), \( BC (\Psi_1 - \Psi_2 = \pi = 0) \), \( CD (\Psi_1 + \Psi_2 + 2\pi = 0) \), \( AD \) and \( \Delta \phi - \pi \leq -x, x \leq \Delta \phi \).

\[
\Pr(C|c, l) = \int_{-\Delta \phi /2}^{\Delta \phi /2} \sum_{x=-1}^{1} \int_{-\pi}^{\pi} p_{\Psi_1}(y) dy + \int_{-\Delta \phi /2}^{\Delta \phi /2} \int_{-\pi}^{\pi} p_{\Psi_2}(y) dy.
\]

The second integral of each term in (15) can be simplified using the \( F \) function introduced in [5]. For example,

\[
\int_{-\pi}^{\pi} p_{\Psi_2}(y) dy = \begin{cases} F(x + \pi) - F(-x) + 1 & \text{if } -x \leq \Delta \phi \leq x \pm \pi \\ F(x + \pi) - F(-x) & \text{otherwise} \end{cases}
\]

when \( \Delta \phi - \pi \leq -x, x \leq \Delta \phi \).
Fig. 4 shows an example of the directly calculated FH/CPFSK error probability with limiter-differential detection as a function of $\gamma$, the partial-band jamming fraction, when $L = 2$, $E_b/N_0 = 15$ dB, $h = 0.7$, and $BT = 1$. For $E_b/N_j$ greater than some value, there is a value of $\gamma$ less than one which maximizes the bit error probability. Some example worst-case $\gamma$'s for FH/CPFSK differential detection with a limiter are compared in Table I with those pertaining when a limiter is not used. It is evident for the case illustrated that the worst-case $\gamma$ is a factor of three higher when limiter-differential detection is employed.

The error probability achieved by the FH/CPFSK system with a limiter-differential detection receiver under worst-case partial-band jamming is plotted in Fig. 5 for the case of $E_b/N_0 = 15$ dB, $h = 0.7$, $BT = 1$, and $L = 1$ and 2 hops/bit. Note that the error probability curve for $L = 2$ exhibits a “diversity gain” for $E_b/N_j$ in the region (approximately) of 10 to 42 dB. The maximum gain is about 1.5 dB for $L = 2$ as compared to the no-diversity case ($L = 1$). This amount of gain is not very significant, but does demonstrate that diversity gains against worst-case jamming can be realized for differential detection of hopped CPFSK using simple nonlinear diversity combining techniques.

For the purposes of comparison, the worst-case error probability performance of the differential detection receiver (without the preceding limiter) [10] is plotted in Fig. 6, for the same parameter values as in Fig. 5. What we observe in Fig. 6 is that the differential detector without a limiter produces a consistently higher error probability for a higher number of hops per bit, similar to the results of unweighted linear combining of chips for other modulations. Comparing Figs. 5 and 6, we observe that altogether the use of a preceding bandpass limiter provides as much as a 3 dB gain against the worst-case jammer for $L = 2$.

To provide a further perspective on the amount of performance improvement gained by using a bandpass limiter, we consider a system that uses side information on the jamming state. It is well-known that perfect side information on the jamming state can improve FH/MFSK performance if it is used to deemphasize the jammed hops and thus emphasize the unjammed hops [14, p. 216, vol. 1]. Similarly, one can further improve the performance of the limiter-differential detection receiver beyond the diversity gain shown in Fig. 5 if one assumes the availability of such perfect side information. This information can be used to discard jammed chips from consideration in the

<table>
<thead>
<tr>
<th>$E_b/N_j$ (dB)</th>
<th>Worst-Case $\gamma$ With Limiter</th>
<th>Worst-Case $\gamma$ Without Limiter</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 dB</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>10 dB</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>20 dB</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>30 dB</td>
<td>0.003</td>
<td>0.001</td>
</tr>
</tbody>
</table>
bit decision, unless of course all chips have been jammed, in which case none are discarded. If this procedure is followed, the conditional error probability expression given previously in (10) for \( L = 2 \), when perfect side information is used, becomes:

\[
Pr(E|c) = (1 - \gamma)^2 Pr(\sin \Psi_1 + \sin \Psi_2 < 0|c, l = 0 \text{ hops jammed}) + 2\gamma(1 - \gamma) Pr(\sin \Psi_1 < 0|c, l = 1 \text{ hop jammed}) + \gamma^2 Pr(\sin \Psi_1 + \sin \Psi_2 < 0|c, l = 2 \text{ hops jammed}). \tag{17}
\]

Fig. 7 shows the performance of an FH/CPFSK receiver with perfect jamming side information under the worst-case partial-band jamming condition and the parameter values indicated. Note that more than 10 dB gain can be achieved using side information for \( L = 2 \) when the jamming is strong (low values of \( E_b/N_0 \)).

We observe in Fig. 7 that the receiver with perfect side information has an irreducible error probability for \( L = 2 \), labeled in the figure as "minimum error for \( L = 2 \) and \( E_b/N_0 = 15 \text{ dB} \)." Actually, this kind of irreducible error is observed in any receiver for any \( L \) value, because as \( E_b/N_0 \to \infty \), the system is still subject to additive thermal or background noise and therefore experiences errors without jamming. In addition, we observe that the receiver without perfect side information (Fig. 5) has a much better bit error probability for \( L = 2 \) than the one with perfect side information (Fig. 7) when \( E_b/N_0 > 30 \text{ dB} \); that is, as the jamming becomes very weak. This seemingly contradictory result is explained by the fact that the "perfect" side information was assumed to be used to discard jammed chips, no matter how weakly jammed; this policy works well in the absence of thermal noise. However, with nonzero thermal noise, the error probability is maximized, when the jammer is weak, by a worst-case \( \gamma \) value which makes it highly probable that just one chip is jammed and is discarded, leaving the system with only half the bit energy on which to make a decision in the presence of thermal noise. The limiter-differential receiver without side information, on the other hand, makes use of all the received signal energy.

IV. CONCLUSION

Error probability analyses for the differential detector with a preceding bandpass limiter were presented for \( L = 2 \) diversity, slow-hopping FH/CPFSK waveforms in the worst-case partial-band jamming noise channel. It was shown that soft-decision combining of samples from a differential detector preceded by a limiter can mitigate the jamming effects on the system, producing a diversity gain for error rate improvement. Previous analyses had indicated that soft-decision combining of samples from a dif-

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**Fig. 6.** Worst-case bit error probability versus \( E_b/N_0 \) for FH/CPFSK differential detection without limiter when \( E_b/N_0 = 15 \text{ dB}, \ h = 0.7, BT = 1 \), and the number of hops/bit \( L = 1, 2 \).

**Fig. 7.** Worst-case bit error probability versus \( E_b/N_0 \) for FH/CPFSK limiter-differential detection with perfect jamming state information when \( E_b/N_0 = 15 \text{ dB}, \ h = 0.7, BT = 1 \), and the number of hops/bit \( L = 1, 2 \).
ferential detector without a preceding limiter results in an error probability which increases with the order of diversity. These results indicate that for diversity transmissions under a bit energy constraint, the limiter-differential detection receiver implements a form of nonlinear diversity combining, capable of providing jamming resistance for FH/CPFSK systems.

REFERENCES


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