Complementary Filter Design for Testing of IS-95 Code Division Multiple Access Wireless Communication Systems

Weiguang Hou, Student Member, IEEE and Hyuck M. Kwon, Senior Member, IEEE

Abstract—In this paper, a simple design of a complementary filter is presented for performance measurement of an IS-95 code division multiple access wireless communication system [1]. The complementary filter is required to meet the design requirements, which are specified in IS-97 document [2]. The purpose of employing the complementary filter in test equipment is to remove intersymbol interference introduced by the base station transmit-filter and the transmit-phase equalizer. Five window functions, which do not affect the desirable response at the proper sampling times, are applied to the desirable output. The output of a window is set to be the output of the complementary filter. Then, the complementary filter is obtained by taking a deconvolution of the output response of the window with the impulse response resulted by cascading the base station transmit-filter and the transmit-phase equalizer. It is found that the output of the complementary filter with the Kaiser window function meets all requirements specified in IS-97. In addition, it is observed that the Kaiser window has the smallest number of the filter coefficients among all windows considered in this paper. Therefore, the complementary filter with the Kaiser window is recommended for implementation of the IS-95 system measurement.

Index Terms—CDMA, complementary filters, FIR digital filters, intersymbol interference, Kaiser window.

I. INTRODUCTION

The Telecommunication Industry Association (TIA) specifies various measurement standards to ensure the compatibility of North American code division multiple access (CDMA) cellular transmitters and receivers. The base station (BS) transmit-filter, specified in the IS-95 standard [1, pp. 7–20], introduces intersymbol interference (ISI). Moreover, the transmit-filter must incorporate an all-pass phase equalizer, which produces an asymmetric transmitter impulse response. The ISI introduced by the transmit-filter and the transmit-phase equalizer has negligible effect on the demodulation of the transmit-data. In other words, it is not required to remove the ISI in a mobile station (MS). However, the ISI needs to be removed for the measurement of BS modulation accuracy by employing a complementary filter, whose requirements are specified in IS-97 [2, pp. 6–12]. In other words, the received signal coming from the BS, after down converter and compensation, shall be passed through a complementary filter to remove ISI introduced by the transmit-filter and the transmit-phase equalizer.

Digital-filter designs have extensively been discussed in many literatures [3], [4]. It is relatively well known how to design a filter which removes ISI by using any applicable method. The designed filter may or may not be low-pass. However, the complementary filter, specified in the IS-97, is a low-pass filter and also removes the ISI. Furthermore, its bandwidth is less than 625 kHz. The authors could find no literature about how to design a filter, which both removes ISI and satisfies the specified bandwidth restriction. This paper will demonstrate a way to design a complementary filter which meets the requirements specified in the IS-97. The finite-duration impulse response (FIR) filters are attractive in practice, because they can be stable, efficiently realizable on general- or special-purpose hardware, and can achieve linear phase response. Therefore, only a FIR filter is considered in this paper.

This paper is organized as follows. Section II describes the BS transmit-filter and the transmit-phase equalizer. Section III discusses a method of complementary filter design. Section IV shows a complementary filter design using window functions, and compares different filter designs by using different window functions. In addition a design example is given. Finally, Section V concludes the paper.

II. BASE STATION TRANSMIT-FILTER AND PHASE-EQUALIZER

Fig. 1 shows a simplified base-band equivalent block diagram for a base station testing system. In IS-95, the in-phase (I) and quadrature-phase (Q) impulse sequences after the pseudo noise (PN) spreading operation are fed into the I and Q FIR base-band filters, respectively. Also, the outputs of the I- and Q-FIR base-band filters pass through a phase equalization filter. This phase equalization simplifies the design of the mobile station receiver filters. The phase of receiver filters increases in general as frequency increases, while the phase of the transmitequalizer is decreasing as frequency increases. In other words, the received signal is phase-compensated through down-conversion receiver filters. Then, a complementary filter removes ISI introduced by the base station transmit-filter and the transmit-phase equalizer for measurements.
Let $s(t)$ be the impulse response of the base-band FIR filter. Then $s(t)$ shall satisfy the following equation [1, p. 7–20]:

$$\text{Mean Squared Error} = \sum_{k=0}^{\infty} |s(kT_s) - h(k)|^2 \leq 0.03$$  \hspace{1cm} \text{(1)}$$

where the constants $\tau$ and $\rho$ are used to minimize the mean square error. The constant $T_s$ is equal to 203.451… ns, which equals a quarter of a PN chip $T_c = 1/12288M$. Four samples are employed per chip interval. The values of the coefficients $h(k)$, for $k < 48$, are given in Table I, and $h(k) = 0$ for $k \geq 48$. Note that $h(k)$ equals $h(47 - k)$. In this paper, $s(k)$ is selected to be the same as $h(k)$. For different selections of $s(k)$, the design procedure in this paper remains unchanged.

The phase equalizer in the base station for the transmit signal path has the equivalent base-band transfer function as

$$H_e(\omega) = K\frac{\omega^2 + j\omega\theta - \omega^2_0}{\omega^2 - j\omega\theta - \omega^2_0}$$  \hspace{1cm} \text{(2)}$$

where $K$ is an arbitrary gain, $\theta$ equals $\sqrt{-1}$, $\omega_0$ equals 1.36, and $\omega_0$ equals $2\pi \times 3.15 \times 10^5$ [1, p. 7–20]. By taking the inverse Fourier transform of the transfer function in (2), the impulse response of the phase equalizer can be written as

$$h_e(t) = K\{\delta(t) + h_{e1}(t)\}$$  \hspace{1cm} \text{(3)}$$

where

$$h_{e1}(t) = \frac{2(a + b)}{(a - b)} j[ae^{-bt} - be^{bt}]u(t)$$  \hspace{1cm} \text{(4)}$$

and $u(t)$, $\delta(t)$ are the unit step function and Dirac delta function, respectively. Or $h_{e1}(t)$ in (3) can be written as

$$h_{e1}(t) = \frac{2\theta u(t)}{\sqrt{4 - \theta^2}} e^{-\frac{\sqrt{4 - \theta^2}}{2}t} \left[\theta\cos\left(\frac{\omega_0\sqrt{4 - \theta^2}}{2}t\right) - \omega_0\sqrt{4 - \theta^2} \cos\left(\frac{\omega_0\sqrt{4 - \theta^2}}{2}t\right)\right].$$  \hspace{1cm} \text{(5)}$$

Let $y(n)$ be the impulse response resulted by cascading the base-band FIR transmit-filter and the transmit-equalizer. Then, $y(n)$ can be computed in two ways. One way is by taking a time convolution of the impulse response of the base-band transmit filter $h(n)$ with that of the transmit-equalizer $h_e(n)$ in (3), i.e.,

$$y(n) = h(n) * h_e(n) = h(n) + h(n) * h_{e1}(n)$$  \hspace{1cm} \text{(6)}$$

where $*$ means convolution operation, and $h_{e1}(n) = h_{e1}(nT_s)$. The base-band transmit filter coefficients $h(n)$ are given at a sampling rate $4 \times 12288$ MHz. This sampling rate is not high enough to provide a more accuracy for the discrete time sequence of $h_{e1}(n)$. Thus, $h_{e1}(t)$ is sampled at a higher sampling rate, and $h(n)$ is interpolated. Another way to find $y(n)$ is by finding the frequency response of the cascaded filter, and then taking the inverse Fourier transform of the frequency response of the cascaded filter. In other words

$$Y(\omega) = H(\omega)H_e(\omega)$$  \hspace{1cm} \text{(7)}$$

and

$$y(n) = F^{-1}\{Y(\omega)\}$$  \hspace{1cm} \text{(8)}$$

where $H_e(\omega)$ is given in (2), and $H(\omega)$ is the frequency response of $h(n)$ given by the following equation:

$$H(\omega) = e^{-jM\omega} \sum_{n=0}^{N_s-1} \cos(n(M - n)).$$  \hspace{1cm} \text{(9)}$$

In (9) $M = (N_s - 1)/2$, and $N_s = 48$. The resulting impulse response $y(n)$ is plotted in Fig. 2, which shows an asymmetrical time response.
III. COMPLEMENTARY FILTER

In IS-97, a complementary filter $h_c(n)$ is required to remove the ISI in $y(n)$ due to the base-band filter and equalizer. Let $z(n)$ denote the output response of the complementary filter as

$$z(n) = y(n) * h_c(n). \quad (10)$$

The output response $z(n)$ shall approximately satisfy Nyquist’s criterion for zero ISI. The null levels of $z(n)$ at the sample times shall be at least 50 dB below the on-time response of $z(n)$. In addition, the noise bandwidth of the complementary low pass filter shall be less than 625 kHz [2, pp. 6–12]. If the requirement is only for removing the ISI, it is not difficult to design such a filter to achieve that requirement. For example, an FIR filter $h_c(n)$ with $N_c$ taps, for which the impulse response of $z(n)$ is almost zero at the sample times, may be designed. However, the designed filter may not be a low-pass filter. Let alone it may not meet the bandwidth requirements. The problem is to design a filter which satisfies the requirement both in time and frequency domain.

Let $Y(\omega)$ be the frequency response of $y(n)$, which is a low pass. If $H_c(\omega)$ is a low pass, then the overall filter output, $Z(\omega) = Y(\omega)H_c(\omega)$, resulted by cascading the base-band transmit filter, the transmit equalizer, and the complementary filter, is also a low-pass filter, and satisfies Nyquist’s criterion. A raised-cosine (RC) filter is a low-pass filter and satisfies the Nyquist’s sampling criterion in time domain. Using an RC filter for the desired output $z(n)$ with a proper roll-off parameter $\alpha$, $z(n)$ can be rewritten as shown at the bottom of the page where $m$ equals $(N - 1)/2$, $N$ is odd, and $d$ equals 4 because four samples are used per one $z(n)$. An RC function yields a linear phase filter, which is always a desirable character for filter designs. To find $h_c(n)$ in (11), a deconvolution of $z(n)$ with $y(n)$ is taken as

$$h_c(n) = z(n) * y(n)^{-1} \quad (13)$$

where $*^{-1}$ means deconvolution operation. Let $N_c$ and $N$ be the number of taps of the complementary filter and the resulted RC filter, respectively. And let $N_e$ be the number of the taps of the filter $y(n)$ by cascading the base-band filter $h(n)$ and the equalizer $h_e(n)$. A relation among $N$, $N_c$, and $N_e$ holds as $N = N_c + N_e - 1$. The overall filter output in frequency domain $Z(\omega) = Y(\omega)H_c(\omega)$ may be rewritten as

$$H(\omega)H_e(\omega)H_c(\omega) = Z(\omega). \quad (14)$$

It seems to be correct to state that the frequency response of the complementary filter $H_c(\omega)$ may be

$$H_c(\omega) = \frac{Z(\omega)}{H(\omega)H_e(\omega)}. \quad (15)$$

from (14). However, this may be impossible because $H(\omega)$ in (15) may be equal to zero at some high frequencies.

A complementary filter needs at least 100 tap coefficients to meet the requirements. The more coefficients, the higher the price for implementation and also the longer the delay. This is not desirable for filter designs even though the complementary filter may meet the requirements. Window functions are frequently used in filter designs to make the number of coefficients small [3], [4]. A complementary filter with fewer coefficients can be designed by applying window functions to the RC function.

IV. WINDOWING FUNCTIONS AND EXAMPLES

A truncation of the infinite or length-$L$ time-domain sequence causes an oscillatory behavior at a discontinuity in the frequency domain, called the Gibbs phenomenon. This truncation can be viewed as multiplying the prototype time-domain sequence by a rectangular function that has value unity for $-M \leq n \leq M$ and zero outside that range. In the filter design, a window function is often applied to the original desired sequence to reduce the Gibbs phenomenon. Standard windows can be found in [3] and [4] as shown below.
TABLE II
GENERALIZED COSINE WINDOWS AND THEIR PARAMETERS $a$, $b$, and $c$

<table>
<thead>
<tr>
<th>Window Type</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Hanning</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>Hamming</td>
<td>0.54</td>
<td>0.46</td>
<td>0</td>
</tr>
<tr>
<td>Blackman</td>
<td>0.42</td>
<td>0.5</td>
<td>0.08</td>
</tr>
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</table>

A. Bartlett Triangular Window

$$w(n) = \begin{cases} \frac{2^{n+1} - 2}{N+1}, & n = 0, 1, 2, \ldots, \frac{N-1}{2}, \\ \frac{2^{n+1}}{N+1}, & n = \frac{N-1}{2}, \ldots, N-1, \\ 0, & \text{otherwise.} \end{cases} \quad (16)$$

B. Generalized Cosine Windows

The generalized cosine windows can be written as shown in (17), at the bottom of the page, where the parameters $a$, $b$, and $c$ determine the window type. Rectangular, Hanning, Hamming, and Blackman windows are commonly used cosine windows. The parameters for these four windows are listed in Table II.

C. Kaiser Windows

The Kaiser window with parameter $\beta$ can be written as

$$w(n) = \begin{cases} I_0(\beta \sqrt{1-(\frac{2n}{N})^2}), & n = 0, 1, 2, \ldots, N-1, \\ 0, & \text{otherwise} \end{cases} \quad (18)$$

where $I_0(x)$ is the modified Bessel function of order zero.

The rectangular window function in (17) is the most straightforward. The Bartlett or triangular window reduces the overshoot but spreads the transition region considerably. The Hanning, Hamming, and Blackman windows use progressively more complicated cosine functions to provide a smooth truncation of the ideal impulse response and a frequency response that looks progressively better. The Kaiser window is probably the best because parameter $\beta$ allows adjustment of the compromise between the overshoot reduction and transition region width spreading.

The complementary filter $h_c(n)$ is obtained by applying a window function $w(n)$ to the desired output response $z(n)$ in (11) as follows:

$$y(n) \ast h_c(n) = z(n)w(n). \quad (19)$$

If $z(n)$ satisfies the Nyquist’ s criterion, i.e., $z(n)$ approximately equals zero at sample times, then $z(n)w(n)$ will also satisfy Nyquist’s criterion, i.e., $z(n)w(n)$ will approximately equal zero at sample times. So the window function will only affect the frequency response of the resulted filter, hence the complementary filter frequency response. With proper window parameters, a complementary filter can be designed by simply taking a de-convolution of $z(n)w(n)$ with $y(n)$ in (19). In other words,

$$h_c(n) = (z(n)w(n))^{-1} y(n). \quad (20)$$

Table III shows some design examples for comparison. The $\alpha$ in Table III is the roll-off parameter for the RC function $z(n)$ in (12). All of the filters are low-pass with bandwidth less than 625 kHz and stop-band attenuation greater than 20 dB. The stop-band attenuation of the complementary filter is not specified in IS-97. For the sake of comparison, the stop-band attenuation is set to 20 dB. If higher attenuation is needed, the filter generally will need more coefficients. The Bartlett window function is not applicable in this design. Even with more than one thousand taps, the complementary filter still cannot meet the requirements. So, the Bartlett window is not included in Table III. From the results in Table III, it can be seen that the Kaiser window is the best in terms of the length of the filter coefficients.

Fig. 3 shows the corresponding time impulse response of the complementary filter using the Kaiser window in Table III. Table IV lists the coefficients of the complementary filter with 42 taps by using the Kaiser window function in Table III.
**TABLE IV**

<table>
<thead>
<tr>
<th>K</th>
<th>h₀(k)</th>
<th>h₂(k)</th>
<th>h₄(k)</th>
<th>h₆(k)</th>
<th>h₈(k)</th>
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</table>

**Fig. 4.** Frequency response $H_c(f)$ of the complementary filter using Kaiser window in Table III.

Fig. 4 shows the corresponding frequency response of the filter. The gain factor $K$ in (2) is selected to be 1. The passband edge frequency is about 560 kHz, i.e., the bandwidth of the low-pass filter is about 560 kHz, which is less than 625 kHz. The maximum pass-band ripple of the filter is about 2.2 dB. The minimum stop-band attenuation is greater than 20 dB.

**V. CONCLUSION**

A simple design method for a complementary filter, which satisfies both time and frequency response constraints specified in IS-97 to test accuracy of an IS-95 code division multiple access base station transmitter, was demonstrated. Several complementary filters were obtained and compared by applying different window functions to the desirable output response and by using de-convolution techniques. The complementary filter obtained by using the Kaiser window function was the best among them in terms of the length of the filter taps. This complementary filter with the Kaiser window also removes the ISI introduced by the base station transmit-filter and the transmit-phase equalizer. In addition, this complementary filter meets the noise bandwidth constraint specified in IS-97. Therefore, the complementary filter in this paper can be applicable for testing the IS-95 wide band cellular spread spectrum wireless communication systems.

**REFERENCES**


Weiguang Hou (S’99) received the B.S.E.E. and M.S.E.E. degrees from Xidian University (previously The Northwest Telecommunications Engineering Institute), Xi’an, China, in 1983 and 1989, respectively. He also received the M.S.E.E. from Wichita State University, Wichita, KS, in 1996, where he is currently pursuing the Ph.D. degree.

He was an Electronics Instructor with The Secondary Artillery Institute, PLA, Wuhan, China, from 1983 to 1986. He was an Engineer with Guangzhou Broadcasting and TV Bureau, Guangzhou, China, from 1990 to 1993, engaged in TV transmission. He was with IFR Systems, Inc., Wichita, KS, from 1996 to 1998 as a Design Engineer where he developed DSP algorithm for wireless communication test set. He is an Engineer with LP Technologies Inc., Wichita. His research interests include performance analysis of digital communication systems, DSP algorithm development, and smart antenna applications.

Hyuck M. Kwon (S’82–M’84–SM’96) was born in Korea on May 9, 1953. He received the B.S. and M.S. degrees in electrical engineering from Seoul National University, Seoul, Korea, in 1978 and 1980, respectively, and the Ph.D. degree in computer, information, and control engineering from the University of Michigan, Ann Arbor, in 1984.

From 1985 to 1989, he was with the University of Wisconsin, Milwaukee, as an Assistant Professor in the Electronic Engineering and Computer Science departments. From 1989 to 1993, he was with the Lockheed Engineering and Sciences Company, Houston, TX, as a Principal Engineer, working for NASA space shuttle and space station satellite communication systems. Since 1993, he has been with the Electronic Engineering Department, Wichita State University, Wichita, KS, as a faculty member. In addition, he held several visiting and consulting positions at communication system industries, and a Visiting Associate Professor position at Texas A&M University, College Station, in 1997. His current research interests are in wireless, code division multiple access (CDMA) spread spectrum, and smart antenna communication systems.