Decision Feedback Postprocessor for Zero-Crossing Digital FM Demodulator

Kwang Bok (Ed) Lee, Member, IEEE, Weiguang Hou, Student Member, IEEE, and Hyuck M. Kwon, Senior Member, IEEE

Abstract—A baseband digital narrow-band FM receiver, called zero-intermediate frequency zero-crossing demodulator (ZIFZCD), has been recently developed. This demodulator may offer low complexity and simple implementation. However, the bit error rate (BER) performance of the ZIFZCD is inferior to that of the limiter-discriminator-integrate-and-dump (LDI) demodulator. In this paper, a simple decision feedback postprocessor (DFP) is proposed to improve the performance of the ZIFZCD. Analysis and simulation BER results of the ZIFZCD with the DFP are presented for minimum-shift keying (MSK) and Gaussian MSK (GMSK) signals under additive white Gaussian noise (AWGN) and mobile fading environments.

Index Terms—GMSK, narrow-band digital FM, wireless communications, zero-crossing demodulation, zero-IF.

I. INTRODUCTION

MANY worldwide digital wireless communication systems such CT2 and DECT are based on narrow-band digital frequency modulation (FM) called continuous-phase frequency-shift keying (CPFSK) [1], [2]. An efficient design of a wireless communication receiver for these narrow-band digital FM signals has become an important issue. Traditionally, a superheterodyne architecture, where channel filtering and demodulation are performed in intermediate frequencies (IF) such as 10.7 MHz, has been employed for receiver implementation [3]. This architecture typically requires two or three frequency down conversions and a crystal channel selection IF filter which is usually expensive, not possible to be integrated in integrated circuits, and has fixed bandwidth. To overcome the drawbacks of a superheterodyne architecture, a direct conversion radio has been investigated [3]. In this architecture, the carrier frequency is down converted to zero intermediate frequency (ZIF) in only one step, and both channel filtering and demodulation are performed in zero intermediate-frequency. As a result, the use of this architecture reduces the number of frequency down conversions and offers the easy integration of digital logic flip-flops, and counters. Thus, the ZIFZCD is simple to implement. However, the ZIFZCD does not perform as well as a conventional limiter-discriminator integrate and dump (LDI) receiver.

The objectives of this paper are to introduce a simple postprocessor to improve performance, and to present the performance analysis of the ZIFZCD with this postprocessor. This postprocessor is based on a decision feedback technique, and requires only one comparator and one flip-flop. The performance of the ZIFZCD with the decision feedback postprocessor (DFP) is investigated through analysis and simulation for MSK and GMSK signals under additive white Gaussian noise (AWGN) static and mobile Doppler frequency shifted fading environments.

Section II describes the system model and briefly reviews the ZIFZCD demodulator. Section III introduces the DFP. Section IV presents the BER analysis of the ZIFZCD with the DFP when MSK signals are transmitted under AWGN environments. Section V shows simulation BER results for MSK and GMSK signals and compares them with analysis BER results. Finally, Section VI concludes the paper.

II. SYSTEM MODEL AND REVIEW OF ZIFZCD

A. System Model

The transmitted signal \( s(t) \) for MSK and GMSK systems may be written as

\[
s(t) = \sqrt{2P} \cos \left( 2\pi f_c t + 2\pi f_d \int_{-\infty}^{t} d(\lambda) \, d\lambda \right)
\]

where \( P \) is the signal power, \( f_c \) the carrier frequency, and \( f_d \) the frequency deviation. For MSK, \( d(t) \) is a \( \pm 1 \) binary data sequence waveform, whereas for GMSK \( d(t) \) is a data sequence waveform modified by the premodulation Gaussian filter. The output signals of a zero-IF down converter in Fig. 1 are the in-phase \( \hat{i}(t) \) and quadrature-phase \( \hat{q}(t) \) components of a hard-limited baseband CPFSK signal. They can be written as

\[
\hat{i}(t) = \cos(\phi(t) + \eta(t) + \delta(t) + \phi_0)
\]

\[
\hat{q}(t) = \sin(\phi(t) + \eta(t) + \delta(t) + \phi_0)
\]

\[\text{(2)}\]

\[\text{(3)}\]
where $\phi(t)$ is the signal phase distorted by the IF filter which causes intersymbol interference (ISI) effects, $\eta(t)$ is the signal-dependent phase noise due to the AWGN, $\delta(t)$ is the phase noise due to the fading, and $\phi_0$ is the initial phase of the signal. Under the pure AWGN environment, $\delta(t)$ is zero.

A receiver estimates a transmitted data symbol based on a measured phase rotation angle $\Delta \phi$ over one symbol period, that consists of the signal phase change $\Delta \phi$ and the phase noise change $\Delta \eta$ [6]. $\Delta \phi = \phi(t) - \phi(t-T)$ and $\Delta \eta = \eta(t) - \eta(t-T)$. Note that $\delta(t)$ is assumed to change slowly so that $\Delta \delta = \delta(t) - \delta(t-T) = 0$.

### B. ZIFZCD

The ZIFZCD receiver in Fig. 1 estimates a phase rotation angle over one symbol time by counting the number of times that the $I$ and $Q$ axes are crossed by the phase trajectory in the phase diagram [5]. The phase axes crossing occurs when the $i(t)$ and $q(t)$ signals cross the zero axis in the time domain.

The ZIFZCD consists of a phase axis generator, a zero-crossing detector, a zero-crossing counter, and a symbol decision device. The phase axis generator is used to generate additional phase axes to estimate a phase rotation angle for MSK and GMSK signals. This is done by adding and subtracting $i(t)$ and $q(t)$ signals. In this paper, the total number of axes is assumed to be $L$. A zero-crossing detector detects zero crossings (i.e., phase axis crossings) and determines phase-rotation direction.

The zero-crossing counter counts the total number of zero crossings over a symbol time interval. This number is related to the total phase rotation angle over one symbol duration, and is used to estimate a transmitted data symbol by the symbol decision device. Symbol synchronization is assumed to be perfect [7].

### III. DECISION FEEDBACK POSTPROCESSOR

An alternating data bit pattern, e.g., 1 1 1 0 0 1 0, produces the smallest phase change $\Delta \phi$ for the second symbol interval in the data pattern due to the intersymbol interference (ISI). 1 and 0, respectively, represents 1 and 0 bits. The small phase change may result in no zero-crossing detection by the ZIFZCD. The probability of no zero-crossing detection decrease as the number of phase-axes $L$ increases. However, $L$ should be minimized to reduce implementation complexity. $L$ has to be greater than and equal to four, because the maximum phase change is $\pi/2$ in MSK.

When no zero crossing is observed for a symbol time interval, a transmit data symbol may be randomly estimated as one or zero or estimated as the opposite of a previous bit estimation. The inversion of the previous bit for no zero-crossing case may be called a DFP. This postprocessing technique has the effect of changing adaptively decision thresholds according to bit patterns. For example, when $L$ is four and the phase angle at the beginning of the second bit time is $\pi/8$, the decision threshold for the second bit is $\pi/8$ instead of 0 for bit patterns 11 and 1 1. Similarly, the decision threshold for the second bit is $-\pi/8$ instead of 0 for bit patterns 1 1 and 1 1. This decision feedback postprocessing exploits the property that the second bit in an alternating data bit pattern produces much less phase change due to the ISI than the second bit in a same data bit pattern. This postprocessing is found in Section IV to significantly improve the BER performance, compared to the random decision scheme whenever no zero crossings occur.

The implementation complexity of the DFP is insignificant, since it requires one 1-bit comparator and one 1-bit register to store a previous bit. This DFP is much simpler than a multiple level decision postprocessing technique in [8], which requires multiple-bit A/D converters and a multiple number of multiple-bit comparators.

### IV. BER ANALYSIS OF ZIFZCD WITH DFP

Since decisions made by the ZIFZCD with the DFP on two consecutive symbols are statistically related, data sequences of two symbols are considered for performance analysis: 11, 01, 00, and 10. They are assumed equally probable and denoted by $\alpha_i$, $i = 1, 2, 3, 4$, respectively. The signal phase change $(\Delta \phi_1, \Delta \phi_2)$ for two consecutive symbol time intervals may be precalculated for a given transmitted data pattern $\alpha_i$ by using Pawula’s analysis in [6]; a sequence is represented by
where \( \lambda \) denotes the transpose of a vector and \( C^{-1} \) is the inverse of the covariance matrix. Let \( y_1 \) and \( y_2 \) denote, respectively, \( y_1 = \Delta \eta_1 \) and \( y_2 = \Delta \eta_1 + \Delta \eta_2 \). Then, \( \Delta \eta_1 = y_1 \) and \( \Delta \eta_2 = y_2 - y_1 \), and the joint PDF of \( y_1 \) and \( y_2 \) can be written using (6) as

\[
p_{y_1 y_2}(y_1, y_2) = \frac{1}{2 \pi \sigma^2} \exp \left\{ -\frac{1}{2 \sigma^2} (y_1^2 + y_2^2 - 2y_1y_2) \right\}
\]

The numbers of zero crossings during previous and current symbol time intervals are respectively denoted by \( n_1 \) and \( n_2 \). They are typically in the range of \((-L, L)\), when the number of phase axes is equal to \( L \). There are two data patterns “11” or “01” with current bit “1,” denoted by \( a_1 \) and \( a_2 \), respectively. The current bit decision will be incorrect if \( n_2 \) is zero and \( n_1 \) is positive because the DFP will flip the previous bit decision. In other words, an incorrect bit decision will be made if \( \{n_1 < 0 \cap n_2 < 0\} \) or \( \{n_1 > 0 \cap n_2 < 0\} \) for \( a_1 \) and \( a_2 \) data pattern transmissions. It is assumed that \( Pr(n_1 = 0, n_2 = 0) \approx 0 \), since the probability that no zero crossings occur over a consecutive two-bit time interval is negligible.

In a digital FM receiver, clicks may occur when SNR is small [6]. Clicks cause a multiple of 2\( \pi \) phase rotation in the opposite direction of signal phase rotation. In the BER analysis, we will first consider no click noise case, followed by a click noise case. Suppose that no click occurs for the previous and current bit intervals. The conditional BER, given one bit transmission, will be the average of the conditional BER over patterns \( a_1 \) and \( a_2 \)

\[
Pr[E\mid '1', \phi_0] = Pr[E\mid a_1, '1', \phi_0]Pr(a_1) + Pr[E\mid a_2, '1', \phi_0]Pr(a_2)
\]

(9)

because \( Pr(a_1) = Pr(a_2) = 1/2 \). The conditional BER given \( a_i \) for \( i = 1 \) and 2 can be written as

\[
Pr[E\mid a_i, '1', \phi_0] = Pr[n_1 < 0 \cap n_2 < 0]Pr(a_i, \phi_0) + Pr[n_1 > 0 \cap n_2 < 0]Pr(a_i, \phi_0)
\]

(10)

Since \( Pr(E\mid \phi_0) = Pr(E\mid '1', \phi_0) \), the overall BER is calculated by averaging \( Pr(E\mid '1', \phi_0) \) over \( \phi_0 \) in \([0, \pi/L]\)

\[
Pr(E) = \frac{1}{\pi} \int_0^{\pi/L} Pr(E\mid '1' \text{ bit, } \phi_0) d\phi_0.
\]

(11)
given a data pattern transmission, can be written as

\[ \Pr(n_1 < 0 \cap n_2 < 0|a_i, \phi_0) = \sum_{i=-L}^{1} \int_{b_1}^{c_1} \int_{-\infty}^{c_2} p_2(y_1, y_2) \, dy_1 \, dy_2 \]  

where \( p_2(y_1, y_2) \) is given in (7). The BER of the ZIFZCD with the DFP with no click can be computed by using (9)–(13).

The conditional bit error probability given no click, \( \Pr(N_1 = 0, N_2 = 0) \), has been found in (9)–(13). We assume \( N_1 \) and \( N_2 \) are discrete and independent random variables with a Poisson distribution [6] as

\[ P(N = k) = \left( e^{-\overline{N}} \overline{N}^k / k! \right) \]  

where \( \overline{N} \) is the average number of clicks for a symbol time interval. Then, \( \Pr(N_1 = 0, N_2 = 0) = e^{-\overline{N}_1} \overline{N}_1^0 / 0! \), \( \Pr(N_1 = 1, N_2 = 0) = \overline{N}_1 e^{-\overline{N}_1} / 1! \), and \( \Pr(N_1 = 0, N_2 = 1) = \overline{N}_2 e^{-\overline{N}_2} / 1! \) are determined. \( \overline{N}_1 \) and \( \overline{N}_2 \) can be calculated using (21)–(26) in [6] for all data patterns.

The conditional probability \( \Pr(n_2 \leq 0|N_1 \geq 1, N_2 = 0) \) in (14) can be calculated as

\[ \Pr(n_2 \leq 0|N_1 \geq 1, N_2 = 0) = \frac{L}{\pi} \int_{0}^{\pi/L} \int_{-\Delta\phi_2}^{\Delta\phi_2} p_\eta(\eta) \, d\eta \, d\phi_0 \]  

where \( p_\eta(\eta) \) is the Gaussian PDF of the differential phase with variance \( 1/\gamma \) [9]. By using (14), (15), and the click probability equations, the BER of the ZIFZCD with the DFP, including click noise, can be computed for AWGN channels.

V. NUMERICAL RESULTS

Fig. 3 shows simulation and analysis BER results for the LDIF, and the ZIFZCD with and without the DFP, when \( L = 4 \) phase axes are employed. MSK signals are transmitted under AWGN environments. The receiver IF filter used is a Gaussian filter with the bandwidth time product \( B_r T_b \) equal to one. It is observed that at 1% BER the ZIFZCD with the DFP is 3.6 dB better than the ZIFZCD without postprocessing, and the ZIFZCD with the DFP is about 1 dB worse than the LDIF.

Additional performance analysis and simulations with \( L = 8 \) show that the ZIFZCD with the DFP and eight axes is 1.7 dB better than the ZIFZCD with the DFP and four axes, and 5.3 dB better than the ZIFZCD with four axes and no postprocessing at 1% BER. This improvement completely makes up for the difference between the ZIFZCD and the conventional LDIF.

Due to performance analysis complexity, only simulation BER results are presented for fading environments in this paper. The Jake fading model [10] is employed with the Doppler frequency time product \( f_d T_b = 0.0035 \), with \( L = 4 \) phase axes are employed. MSK signals are transmitted under AWGN environments. The receiver IF filter used is a Gaussian filter with the bandwidth time product \( B_r T_b \) equal to one. It is observed that at 1% BER the ZIFZCD with the DFP is 3.6 dB better than the ZIFZCD without postprocessing, and the ZIFZCD with the DFP is about 1 dB worse than the LDIF.

Fig. 4 shows simulation BER results of the LDIF and ZIFZCD with four axes for MSK and GMSK signals in fading environments. For GMSK signals, the transmit bandwidth time product \( B_r T_b \) is set to 0.5. It is observed for MSK signals that the ZIFZCD with the DFP is better than the ZIFZCD without
the DFP by 1.5 dB at 1% BER and only 0.5 dB worse than the LDI. For GMSK signals, the ZIFZCD with the DFP is found to perform as well as the LDI.

**VI. CONCLUSIONS**

A DFP scheme was proposed to improve the performance of the ZIFZCD. The bit error rates (BER’s) of the conventional LDI and ZIFZCD demodulators with and without the DFP were analyzed for MSK signals in AWGN channels. In addition, the LDI and ZIFZCD with and without the DFP were simulated for the demodulation of MSK and GMSK signals in mobile fading channels.

The DFP has been found to improve significantly the performance of the ZIFZCD for MSK and GMSK signals. The DFP reduces the required SNR for 1% BER by 3.6 dB when the ZIFZCD with four axes is employed for MSK signals in AWGN environments. The ZIFZCD with the DFP and four axes has been found to demodulate GMSK signals as well as the LDI in fading environments.

In summary, the ZIFZCD with the DFP is a good alternative to the LDI.

**REFERENCES**


Kwang Bok (Ed) Lee (M’96) received the B.A.Sc. and M.S.E.E. degrees from the University of Toronto, Toronto, Ont., Canada, in 1982 and 1986, respectively, and the Ph.D. degree from McMaster University, Canada, in 1990.

He was with Motorola Canada from 1982 to 1985 and Motorola USA from 1990 to 1996 as a Senior Staff Engineer. At Motorola, he was involved in the research and development of various communication systems. He was with Bell-Northern Research, Canada, from 1989 to 1990. In March 1996, he joined the School of Electrical Engineering, Seoul National University, Seoul, Korea as an Assistant Professor. He has served as a Consultant to a number of industries in communications. His research interests include mobile communications, communication theories, and adaptive signal processing.

Weiguang Hou (S’95) received the B.S.E.E. and M.S.E.E. degrees from Xidian University (previously named The Northwest Telecommunications Engineering Institute) Xi’an, China, in 1983 and 1986, respectively, and the M.S.E.E. degree from Wichita State University, Wichita, Kansas, in 1996. He is currently working toward the Ph.D. degree at Wichita State University.

He was an Instructor at the Secondary Artillery Institute, PLA, Wuhan, China, from 1983 to 1986, where he taught electronics. He was an Engineer with the Guangzhou Broadcasting and TV Bureau, Guangzhou, China, from 1990 to 1993, where he was engaged in TV transmission. He was with IFR Systems, Inc., Wichita, KS, as a Design Engineer from 1996 to 1998, where he developed DSP algorithms for a wireless communication test set. He is an Engineer with LP Technologies, Inc., Wichita. His research interests include performance analysis of digital communication systems, DSP algorithm development, and smart antenna applications.

Hyuck M. Kwon (S’82–M’84–SM’96) was born in Korea on May 9, 1953. He received the B.S. and M.S. degrees in electrical engineering from Seoul National University, Seoul, Korea, in 1978 and 1980, respectively, and the Ph.D. degree in computer, information, and control engineering from the University of Michigan at Ann Arbor, in 1984.

From 1985 to 1989, he was with the University of Wisconsin, Milwaukee, as an Assistant Professor in the Electrical Engineering and Computer Science Department. From 1989 to 1993, he was with the Lockheed Engineering and Sciences Company, Houston, TX, as a Principal Engineer working for NASA space shuttle and space station satellite communication systems. Since 1993, he has been with the Electrical Engineering Department, Wichita State University, Wichita, KS, as a Faculty Member. In addition, he has held several visiting and consulting positions at communication system industries. He was also a Visiting Associate Professor at Texas A&M University, College Station, in 1997. His current research interests are in wireless, CDMA spread spectrum, and smart antenna communication systems.