Optimal Bayesian Data Fusion and Low-complexity Approximations for Distributed DS-CDMA Wireless Sensor Networks in Rayleigh Fading

Sudharman K. Jayaweera
Department of Electrical and Computer Engineering
Wichita State University, Wichita, KS 67260, USA.
Email: sudharman.jayaweera@wichita.edu

Abstract—In this paper we propose non-orthogonal communication between sensors and a data fusion center via direct-sequence code-division multiple-access (DS-CDMA) and investigate the fusion performance in the presence of channel errors due to both multiple-access interference (MAI) and noise. We derive the optimal Bayesian data fusion receiver for such a coherent DS-CDMA based distributed wireless sensor network having a parallel architecture in the presence of Rayleigh fading. It is shown that the complexity of the optimal rule is exponential in the number of local sensors. To provide a low-complexity solution, a class of conceptually simple data fusion receivers that partitions the multi-sensor detection and data fusion into two separate stages are proposed. Several well-known detector structures (joint maximum likelihood (JML), conventional, decorrelator and linear minimum-mean squared error (MMSE)) are considered for the multi-sensor detector at the first stage. The second stage of these receivers perform Bayesian data fusion based on the estimated symbols from the first stage. The performance results indicate that while the conventional detector first stage based receiver performs remarkably close to the optimal fusion receiver in an AWGN noise channel, its performance severely degrades in the presence of Rayleigh fading. In terms of the complexity and performance trade-off, the MMSE first stage based receiver seems to be a good design choice over a wide-range of parameters. It is also observed that the fusion performance is not limited by the channel signal-to-noise ratio (SNR) as long as it is not too small. On the other hand, for large channel SNR values the optimal fusion performance is limited by the quality of local sensor decisions (or, equivalently, the local SNR).

I. INTRODUCTION

Most of current literature on low-power wireless sensor networks assumes orthogonal sensor communication. However, in some applications involving dense, low-power, distributed wireless sensor networks it may be more effective to employ non-orthogonal multi-sensor communication. Direct-sequence code-division multiple-access (DS-CDMA) can be a good candidate for communication between data collection sensors and a fusion center (or a data gathering node) in such sensor networks. Although there are concerns on their power efficiency, such spread spectrum techniques have already been considered for wireless sensor networks in [1], [2].

The relaying of only the local distributed decisions, as opposed to directly sending the sensor observations, to a central fusion center can significantly save system resources in terms of communication bandwidth and sensor power. The two interwind problems in this context of distributed detection and data fusion are both classical problems that date back to 1980’s [3]–[5]. In particular, it has been shown that in a Bayesian approach the solution to the distributed detection problem is a set of likelihood ratio based decision rules at local sensors but with possibly coupled thresholds. The data fusion problem for such a distributed detection system can also be formulated as a Bayesian hypothesis testing problem. The optimal Bayesian data fusion rule was derived in [5] assuming error-free reception of local decisions at the fusion center.

In this paper, we consider Bayesian data fusion based on distributed local decisions in a coherent DS-CDMA wireless sensor network in Rayleigh fading. This model may be applicable in, for example, dense wireless sensor networks with energy-constrained nodes. Although each sensor may have data to be sent to a data fusion node very rarely, due to the large number of nodes in a dense network, multiple nodes may have data at the same time. In such cases, a CDMA based scheme may allow all nodes to access the channel simultaneously and go to an energy saving idle or sleep mode quickly rather than waiting for a long time in an active mode as in a time-division multiple-access (TDMA) based orthogonal signalling scheme.

We first derive the optimal Bayesian data fusion receiver for such a DS-CDMA based distributed wireless sensor network and show that it has a complexity exponential in the number of sensors. In order to reduce this complexity and to provide practical solutions for low-power sensor networks, next we consider low-complexity, sub-optimal fusion receiver
architectures. We propose a class of partitioned receivers in which the multi-sensor detection and data fusion are separated into two stages. For the multi-sensor detection at the first stage we consider several well-known multiuser detectors namely, joint maximum-likelihood (JML), conventional single-user matched filter, decorrelator and linear minimum-mean squared error (MMSE) detectors. The second stage of the partitioned detectors performs the data fusion based on the output values from the first stage.

This paper makes several new contributions both to the distributed detection and data fusion literature as well as wireless sensor networks. One of the main contributions of the paper is the proposal of non-orthogonal multi-sensor communication for low-power wireless sensor networks performing distributed detection and data fusion. Although there is a considerable amount of previous work on the subject of distributed detection, only a very few of them take into account the effect of channel errors. In particular, to the best of our knowledge, this is the first time non-orthogonal DS-CDMA based sensor communication has considered in this context. With the renewed interest in dense wireless sensor networks such non-orthogonal multi-access communication seems to be a logical choice. Another contribution is the formulation of the problem of Bayesian fusion of distributed decisions for such a sensor network based on non-orthogonal communication and the demonstration of general performance characteristics via numerical results. Note that, in this paper we resort to simulations for performance analysis since obtaining analytical expressions seems to be intractable due to the more practical sensor quantizers employed in our model. Our current work involves somewhat modified local detector models in order to facilitate the analytical performance analysis.

The remainder of the paper is organized as follows: First, in section II we present our system model. In section III we derive the optimal Bayesian data fusion receiver for a DS-CDMA based wireless sensor network. Next, in Section IV we present the basic structure of the partitioned fusion receivers and consider several multi-sensor detectors as candidates for the first stage. Section V presents numerical performance results followed by a discussion. Finally, in Section VI we summarize our conclusions and suggest some future research directions.

II. System Model and Description

We consider a binary hypothesis testing problem in a \( K \)-node wireless sensor network connected to a data fusion center in a distributed parallel architecture [4] (the theory that we develop below can easily be extended to distributed multiple hypothesis testing). Let us denote by \( H_0 \) and \( H_1 \) the null and alternative hypotheses, respectively having corresponding prior probabilities \( P(H_0) = p_0 \) and \( P(H_1) = p_1 \). To be concrete, under the two hypotheses the \( k \)-th local sensor observation \( z_k \), for \( k = 1, \cdots, K \), is assumed to be distributed as,

\[
H_0 : \quad z_k \sim \mathcal{N}(0, \sigma^2)
\]

\[
H_1 : \quad z_k \sim \mathcal{N}(\mu_k, \sigma^2)
\]

where \( \mathcal{N}(\mu, \sigma^2) \) denotes a Gaussian distribution with mean \( \mu \) and variance \( \sigma^2 \). Conditioned on the hypothesis, the local observations are considered to be independent of each other. Each local sensor processes its observations independently to generate a local decision \( u_k \in \{0, 1\} \). Assuming a Bayesian approach, the decision \( u_k \) of the \( k \)-th sensor is computed as

\[
u_k = \begin{cases} 
1 & \text{if } L(z_k) \geq \tau_k \\
0 & \text{if } L(z_k) < \tau_k
\end{cases}
\]

where \( L(z_k) \) is the local likelihood ratio (llr) defined by \( L(z_k) = \frac{p(z_k|H_1)}{p(z_k|H_0)} \) and \( \tau_k \) is the threshold of the likelihood ratio test at the \( k \)-th sensor. In the Bayesian formulation, these local sensor thresholds depend on the prior probabilities and an assumed cost function [6]. Assuming independent local sensor decisions \( \tau_k = \frac{p_h(C_{10} - C_{01})}{p_h(C_{11} - C_{00})} \) where \( C_{ij} \) is the cost incurred by choosing hypothesis \( H_i \) when hypothesis \( H_j \) is true. For the minimum probability of error detection at the local sensors the cost function can be chosen to be uniform [6]. It should be emphasized that while optimal distributed detection scheme may require joint determination of local thresholds above formulation can easily be modified for that case. In either case, the optimal fusion rule only depends on quality of the individual local sensor decisions as long as they are independent.

The local decisions \( u_k \)'s, for \( k = 1, \cdots, K \), are next transmitted to the fusion center over a Rayleigh channel using DS-CDMA in which sensor \( k \) employs a normalized signature waveform \( s_k(t) \) of unit energy. Below we will assume that local sensors take a series of observations \( z_k(i) \) corresponding to a series of true hypothesis denoted by either \( H_0(i) \) or \( H_1(i) \). Assuming a binary phase shift keying (BPSK) system (for simplicity), the binary local decisions \( u_k(i) \)'s, for \( k = 1, \cdots, K \), are first symbol mapped to \( b_k(i) \in \{+1, -1\} \) and then the resultant symbol stream of each sensor \( k \) is modulated using the signature waveform \( s_k(t) \) of that sensor. It is clear that by sending the binary local decisions \( u_k \)'s instead of the local observations \( z_k \)'s, the distributed detection and fusion system can reduce the transmission requirements leading to considerable energy savings in a wireless sensor network.

Assuming symbol synchronism among the distributed sensors (which can be relaxed easily), the complex baseband
received signal at the data fusion center can be written as

\[ r(t) = \sum_{i=0}^{M-1} \sum_{k=1}^{K} A_k b_k(i) s_k(t - iT) + n(t) \]

where \( M \) is the number of symbols per sensor per frame, \( T \) is the symbol interval, \( n(t) \) is the zero-mean complex AWGN receiver noise with variance \( \sigma^2 = N_0 / (N_0 / 2) \) per dimension and \( \{ s_k(t); 0 \leq t \leq T \} \) denotes the normalized signature waveform of the \( k \)-th sensor. The complex coefficients \( A_k \)'s are assumed to be zero-mean complex Gaussian with variance \( \bar{A}_k^2 \). In the following we define the \( k \)-th local sensor signal-to-noise ratio \( SNR_k \) as \( SNR_k = \frac{\mu_k^2}{\sigma^2} \) and the average channel SNR of the \( k \)-th sensor \( SNR_k^{ch} \) as \( SNR_k^{ch} = \frac{\bar{A}_k^2}{\sigma^2} \).

### III. OPTIMAL FUSION RECEIVER FOR A DS-CDMA WIRELESS SENSOR NETWORK

Due to the assumed symbol synchronism, the Bayesian fusion problem can be formulated as one of deciding between \( H_0 \) and \( H_1 \) based on the observed receiver waveform \( \{ r(t) : t \in [iT, (i + 1)T] \} \) in order to minimize a cost function. It can easily be shown that a sufficient statistic for this fusion problem is given by the output of a bank of \( K \)-matched filters each matched to a particular user signature waveform \( s_k(t) \) (similar to in optimal multi-user detection). The vector of matched filter outputs \( y = [y_1, \cdots, y_K]^T \) can then be shown to be given by [7],

\[ y = RA_b + n \quad (2) \]

where \( R \) is the \( K \times K \) normalized cross-correlation matrix of sensor signature waveforms, \( A = \text{diag}(A_1, \cdots, A_K) \) is a diagonal fading matrix and \( n \sim \mathcal{N}_c(0, \sigma^2 R) \) is the \( K \)-vector of complex Gaussian receiver noise with mean 0 and covariance matrix \( \sigma^2 R \). Note that in (2) we have dropped the time index \( i \) since it is irrelevant due to the assumed symbol synchronism among the sensors.

Now we may interpret the data fusion problem for the coherent DS-CDMA wireless sensor network as a binary-hypothesis testing based on the observation vector \( y \) and the fading coefficient matrix \( A \). Thus, it can be shown that the Bayesian optimal fusion rule is given by a likelihood ratio-test (LRT) as below:

\[ L(y) = \frac{p(y|H_1, A)}{p(y|H_0, A)} \quad (3) \]

where \( \tau_F \) is the threshold at the fusion center which depends on the prior probabilities \( p_0 \) and \( p_1 \) and the cost function. Note that, in obtaining the above optimal fusion rule we have assumed that the complex fading coefficient matrix \( A \) is independent of the two hypotheses. It can easily be shown that, in the case of minimum probability of error fusion (uniform cost assignment) and equal \( a \) priori probabilities for the two hypotheses, the threshold for the likelihood ratio fusion is \( \tau_F = 1 \).

Using the received signal model (2) the required likelihood ratio at the fusion center can be expressed as,

\[ L(y) = \frac{\sum_{b \in \{+1,-1\}^K} e^{\frac{1}{2} (b^H A y - \frac{1}{2} b^H A R A^H b) \mu_k^2}} {\sum_{b \in \{+1,-1\}^K} e^{\frac{1}{2} (b^H A y - \frac{1}{2} b^H A R A^H b) \mu_k^2}} \quad \frac{p(b | H_1)}{p(b | H_0)} \quad (3) \]

where the summations are over all possible \( 2^K \) transmit symbol vectors and \( \dagger \) denotes the Hermitian transpose.

Assuming that the local sensor decisions are independent we may compute the conditional probabilities \( p(b | H_i) \), for \( i = 0, 1 \) as,

\[ p(b | H_i) = \prod_{k=1}^{K} p(b_k | H_i) \]

where

\[ p(b_k | H_1) = \begin{cases} 1 - P_{M_k} & \text{if } b_k = +1 \\ P_{M_k} & \text{if } b_k = 0 \end{cases} \]

and

\[ p(b_k | H_0) = \begin{cases} P_{F_k} & \text{if } b_k = +1 \\ 1 - P_{F_k} & \text{if } b_k = 0 \end{cases} \]

with \( P_{F_k} \) and \( P_{M_k} \) representing the false-alarm and miss probabilities of the \( k \)-th sensor, respectively. It is assumed that fusion center knows these false-alarm and miss probabilities. For example, based on the local observation statistics in (1) we may show that these local probabilities are given by

\[ P_{F_k} = Q\left( \tau_k' / \sigma_k \right) \quad \text{and} \quad P_{M_k} = 1 - Q\left( \tau_k' - \mu_k / \sigma_k \right) \quad (4) \]

where \( Q(x) \) denotes the Gaussian tail distribution and the thresholds \( \tau_k' \) are given by \( \tau_k' = \frac{\mu_k}{\sigma_k} \log(\tau_k) + \frac{\mu_k^2}{2} \). The optimal
fusion receiver decisions are then given by

$$\delta_{\text{opt}}(y) = \begin{cases} 1 & \text{if } L(y) \geq \tau_F \\ 0 & \text{if } L(y) < \tau_F \end{cases}$$

where \(L(y)\) is given in (3).

IV. LOW-COMPLEXITY, PARTITIONED FUSION RECEIVERS

If we define the required number of multiplications (NoM) as the time complexity of a receiver, then as can be observed from (3) for the binary problem at hand the complexity of the coherent optimal fusion scheme in a fading channel is exponential in the number of local sensors \(K\). In order to reduce this complexity, in this section we propose a class of sub-optimal fusion receivers which separate the multi-sensor detection and fusion into two stages as shown in Fig. 2. In these partitioned receivers, coherent multi-sensor detection is performed at the first stage as in a traditional multiuser detector in order to estimate the symbols \(b_k\)’s transmitted by the local sensors. The estimated symbol vector \(\hat{b}\) is the input to the second stage of the receiver that receives data fusion. The second stage of the receiver performs data fusion based on these outputs as if they were the true local decisions.

In the following we consider several well-known multiuser detectors as the first stage of the partitioned receivers: namely joint maximum likelihood (JML) detection, conventional single user matched filter and two linear multiuser detectors (decorrelator and minimum mean squared error).

The low-complexity partitioned receiver processing can be explained using the JML multi-sensor detector first stage based receiver as follows: The coherent JML multi-sensor detector at the first stage estimates the symbol vector \(\hat{b}\) by the following estimate which maximizes the joint likelihood function of \(b\) [7]:

$$\hat{b}^{(JML)} = \arg \max_{b \in \{-1, +1\}^K} (2y^\dagger b - b^\dagger R A^\dagger R A b).$$

Next, the second stage of the partitioned receiver performs data fusion assuming that the estimated values \(\hat{b}^{(JML)}\) to be the true independent local decisions. Note that, strictly speaking first stage decisions are not independent (and are erroneous), but we make the above assumption to reduce the complexity at the second stage since our goal in this section is to design good low-complexity receivers. Thus, the likelihood ratio test at the second stage of the receiver is,

$$L(\hat{b}^{(JML)}) = \frac{p(\hat{b}^{(JML)} | H_1)}{p(\hat{b}^{(JML)} | H_0)} = \prod_{k=1}^{K} \frac{p(\mathbf{h}^{(JML)}_k | H_1)}{p(\mathbf{h}^{(JML)}_k | H_0)},$$

where (due to the assumption that the first stage decisions are correct),

$$p(\mathbf{h}^{(JML)}_k | H_1) = \begin{cases} 1 - P_{M_k} & \text{if } \mathbf{h}^{(JML)}_k = +1 \\ P_{M_k} & \text{if } \mathbf{h}^{(JML)}_k = -1 \end{cases},$$

and

$$p(\mathbf{h}^{(JML)}_k | H_0) = \begin{cases} P_{F_k} & \text{if } \mathbf{h}^{(JML)}_k = +1 \\ 1 - P_{F_k} & \text{if } \mathbf{h}^{(JML)}_k = -1 \end{cases}.$$

The required false-alarm and miss probabilities are computed as in (4). Assuming minimum probability of error Bayesian fusion with uniform costs and equal priors, the LRT at the fusion center is then given by,

$$\delta_{JML}(\hat{b}) = \begin{cases} 1 & \text{if } L(\hat{b}^{(JML)}) \geq \tau_F \\ 0 & \text{if } L(\hat{b}^{(JML)}) < \tau_F \end{cases}$$

However, due to the exponential time complexity of the JML multiuser detection at the first stage of the partitioned receiver, still the time complexity for JML-based partitioned fusion receiver is exponential in \(K\). One of the simplest detectors for the multiple-access channel is the single-user matched filter or the so-called conventional detector. The output of the coherent single-user matched filter first stage is given by,

$$\hat{b}^{(MF)} = \text{sgn}(A^\dagger y).$$

While above conventional detector is simple in terms of its complexity, it is known that in the presence of severe multiple-access interference its performance can be very poor compared to the best possible performance. Linear multi-sensor detection first stages can provide a balance between performance and complexity. For example, the coherent decorrelator first stage feeds the following estimates to the fusion block at the second stage:

$$\hat{b}^{(decor)} = \text{sgn}(A^\dagger R^{-1} y).$$

Similarly, we may also employ the so-called linear MMSE multi-sensor detector to obtain first stage estimates as,

$$\hat{b}^{(MMSE)} = \text{sgn}(A^{-1}(R + \sigma^2 A^{-2})^{-1} y).$$
The second stages of all these receivers are exactly the same as that described above for the JML first stage, i.e. the estimated bits from the first stage are used to compute the likelihood ratio as in (6) and the global decision is declared using (7). Note that the complexity of the partitioned receivers are determined by the complexity of the first stage multi-sensor detector since the llr computation (6) in the second stage can be implemented as a table look-up operation.

V. SIMULATION RESULTS

In this section we resort to simulation in order to evaluate the performance of the above data fusion receivers since deriving analytical results seems to be intractable. We investigate the performance and complexity trade-offs of the optimal fusion receiver and the partitioned low-complexity receivers via a numerical example of a 4-node, BPSK-based, synchronous DS-CDMA sensor network in which cross correlation between signature waveforms of each pair of users is assumed to be $\rho = 0.7$ (i.e. a dense network). The Bayesian cost function we consider is the minimum probability of error with equally probable hypotheses. Thus, our performance metric is the probability of error at the output of the fusion receiver.

Figure 3 shows the performance of the fusion receivers as a function of the local SNR for a fixed average channel SNR value $SNR_k^c = 10$dB for all $k$. In Fig. 3a we have first shown the fusion performance of the same system in an AWGN channel. As can be seen from Fig. 3a, the conventional detector first stage provides the best performance out of all partitioned receivers in this case whereas decorrelator first stage results in the worst performance. However, as can be seen from Fig. 3b, the performance of conventional detector based partition receiver severely degrades in a Rayleigh fading channel. In fact, it has the worst performance out of all partitioned receivers. The coherent MMSE first stage based receiver seems to provide the best performance-complexity trade-off in a Rayleigh fading channel.

Another important observation from Fig. 3 is that for a fixed channel SNR the optimal fusion performance monotonically improves as local SNR improves. However, with channel fading the performance of the partitioned receivers may not exhibit this property as can be seen from Fig. 3b. In fact, after a certain local SNR threshold the fusion performance becomes even worse than that of a single sensor. The large performance gap between the optimal fusion and the proposed partitioned receivers suggests that it may be possible to design better low-complexity receivers (note that, the JML first stage may not necessarily provide the best possible fusion performance since it alone has no optimality when employed in a partitioned fusion receiver).

In Fig. 4 we have shown the fusion probability of error performance as a function of the average channel SNR for a fixed local sensor SNR value $SNR_k^l = 10$dB for all $k$. The first conclusion we draw from this figure is that for large channel SNR values, the fusion performance will ultimately be limited by the quality of local sensor decisions (or, equivalently, the local SNR’s). This behavior can be understood more clearly in the context of the partitioned fusion receivers (the same underlying reason holds for the optimal fusion receiver as well). When the channel SNR is low, the first stage is more likely to make erroneous decisions about the transmitted
symbols thereby leading to more fusion errors. As channel SNR improves the first stage decisions becomes better and as a result the final fusion performance also improves as can be seen from Fig. 4 for medium SNR values. However, when the channel SNR is high enough so that the first stage of the partitioned receivers rarely makes an error, the final fusion performance will be as there were no channel errors. In this case, the performance is limited by the accuracy of the local decisions (i.e. local SNR).

The second conclusion is that there is a clear channel SNR threshold below which the fusion performance of all receivers (including the optimal fusion) is worse than that of a single sensor performance. This is to be expected since when there are too many transmission errors fusion of multiple (erroneous) local decisions may not provide any performance gain.

As can be seen from Fig. 4, again the fusion performance of the coherent linear MMSE based partitioned receiver is very close to that achieved by the JML-first stage based receiver but at a greatly reduced complexity. Moreover, for large channel SNR values the performance gap between linear MMSE-based receiver and the optimal fusion receiver can also be very small justifying its use as a good performance-complexity trade-off solution.

VI. CONCLUSIONS AND FUTURE WORK

We considered the problem of Bayesian data fusion based on distributed decisions in a DS-CDMA wireless sensor network in Rayleigh fading. Assuming a binary hypothesis testing problem we first derived the optimal coherent fusion receiver for such a system and noted that its complexity is in general exponential in the number of local sensors. In order to reduce this complexity, we next proposed a class of sub-optimal fusion receivers by partitioning the multi-sensor detection and data fusion into two consecutive stages. For the first stage multi-sensor detection several well known multiuser detectors were considered. Not surprisingly, the JML first stage based receiver was found to perform very close to the optimal fusion receiver in most cases. However, it does not offer any significant complexity reduction since JML multi-sensor detection still has a complexity exponential in the number of sensors. Moreover, although the conventional detector based fusion receiver performs remarkably close to that of optimal fusion receiver in an AWGN noise channel, its performance severely degrades in a coherent Rayleigh fading channel. The linear MMSE first stage based fusion receiver provides a good performance-complexity trade-off and, in particular, provides almost the same performance as that of exponentially complex JML based receiver for high channel SNR values. Moreover, we also observe that the fusion performance is not limited by the channel SNR as long as it is not too small. On the other hand, for large channel SNR values the optimal fusion performance will be limited by the quality of local sensor decisions. The future work will include the analytical performance evaluation of both optimal and proposed partitioned fusion receivers, quantifying the achieved energy efficiencies and extending the theory to asynchronous sensor networks.

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