Improving Revenue Volatility Estimates Using Time-Series Decomposition Methods

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Author Note

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Abstract

Most of the previous research in applied public finance that assesses the effects of revenue volatility has used measures based on the deviation of revenue from its trend growth rate. In this paper, we take a different approach. We rely on the time-series decomposition of the growth rate series to measure volatility of a revenue source. The procedure is shown to produce more accurate measures of volatility that can be used by academics and policy makers. We demonstrate the use of this methodology on two local government revenue sources.

Keywords: Revenue Volatility, Time-Series Methods, Forecasting
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Introduction

The study of the volatility of revenue sources goes back several decades. Early models emphasized measuring the income elasticity of revenue sources, while more recent models examined the time-series volatility of the data. In this paper, we compare the existing models with newer classes of models that may produce cleaner measures of volatility.

Capturing the volatility of revenue is very important for at least two broad reasons. First, from a prospective planning perspective, if we aspire to make decisions about the structure of revenue sources on which we will rely to provide sufficient revenue for essential government services, we should consider not only the mean rate of growth of the revenue sources but also the volatility of those sources in a “portfolio approach” (see for example Garrett 2009; Berg, Marlin and Heydarpour 2000). Second, we can use revenue volatility as an independent variable in developing an understanding about past decisions such as revenue structure, the use of rainy day funds, or state borrowing trends (see Yan 2013; Rodriguez-Tejedo 2012).

In the next section, we review the existing literature on the topic. There have been two dominant approaches taken to measure revenue volatility in the literature. Both of these methods implicitly assume that the long-run trend growth rate of the revenue stream represents the best prediction of the current (and future) revenue realization. We then consider whether that assumption is a good one. The majority of the paper is taken up with an analysis of one traditional measure of volatility and a comparison of the results of that measure with other measures derived from alternative models. We close by considering the empirical results and suggesting other models that may provide superior estimates of volatility.
Literature Review and Discussion

The earliest measures of revenue volatility were based on the concept of income elasticity. In the traditional formulation, this involves regressing the revenue generated (most often in logarithmic form) on personal income for the jurisdiction over which the revenue is generated (again in log form, see Groves and Kahn 1952). More recent formulations have separated long-run processes in the data from short-run, producing markedly different results (see for example Sobel and Holcombe 1996; Bruce, Fox, and Tuttle 2006).

The second method has been to measure deviations of the revenue stream from its underlying trend. This is an easier method computationally, and has been used many times in the applied public finance literature. As formulated by Carroll and Goodman (2011, 85), an exponential trend is generated from the observed data:

\[ y_t = \exp(\beta_0 + \beta_1 t) \]  

(1)

and the residuals from that trend are used as measures of revenue volatility. Most often, the standardized residuals are used by dividing the residual by the predicted value from the regression.

The question should arise whether the results from this most recent set of formulations are efficient measures of volatility. By efficient, we mean that the measure generated by this methodology should capture only random and unpredictable movements of the revenue series. If predictable movements of the time series are captured by the measures, then the residuals will not truly reflect uncertainty or volatility in the data. There is reason to doubt whether models such as shown in equation (1) produce efficient measures of uncertainty. Makridakis and Wheelwright (1989) and Kriz (2012) provide the general form of time series data as consisting of trend, cyclical, seasonal, and error (residual) components. Equation (1) and similar models only
contain information on the trend component. Therefore, if the residuals are interpreted as volatility measures, what the authors of the previous papers that have used these measures have in reality captured is some function of the random error component and the forecastable trend in the data series. In the next section of the paper, we assess to what extent this has affected measurement of revenue volatility.

**Methodology and Data**

In order to assess whether the trend regression methodology was the best model for the growth in revenue and thus is the best method for generating estimates of volatility, we gathered data from the budgets for the city of Omaha, Nebraska for the period 1977 – 2014 and from the Nebraska Department of Revenue for 1975 – 2013. In order to avoid the effect of revenue policy changes, we gathered data on the base of the two most important city of revenue sources, total property valuation and taxable retail sales. Since the data gathered were annual data, we did not have to correct the data for seasonality.

We first generated a trend regression following equation (1). The results of the trend regression along with the original data are shown in Figure 1 for taxable sales and Figure 2 for total property valuation. In general, the fit of the regression is not very good. Specifically, the errors of the regression are likely to be serially correlated, as indicated by the fact that as the forecast for one year has a negative residual, the next year is also likely to have a negative residual (indicating positive serial correlation). We then ran a test for serial correlation on the residuals and found statistically significant positive serial correlation ($\rho_1 = 0.8754$, Box-Ljung Q-statistic = 32.26, $p < .001$ for taxable sales and $\rho_1 = 0.64$, Box-Ljung Q-statistic = 16.83, $p < .001$ for property valuation). What this suggests is that the model is not incorporating enough
information into their predictions, which in turn suggests that the residuals that other researchers have used in their models incorporate not only random variability but also information that is missed in the modeling process.

*Figure 1. Taxable Sales and Predictions from Time Trend Regression, 1975 - 2013.*
In order to assess the efficiency of the time trend regressions, we choose three common alternative models that allow for the decomposition of the time series into trend, cycle, and error. The first model is a simple exponential smoothing filter. We tried many smoothing models. The one producing the best fit was the linear exponential smoothing filter developed by Holt (1957). The forecast model and smoothing equations for this model are:

\[
F_t = a_t + b_t \\
a_t = \alpha X_t + (1 - \alpha)(a_{t-1} + b_{t-1}) \\
b_t = \beta (a_t - a_{t-1}) + (1 - \beta)b_{t-1}
\]  

(2)

The second model was a filter specifically designed to separate a time-series into its components. Again, there are many such filters. We tried several and the one that seemed to fit the data the best was the Hodrick-Prescott (1997) filter. This filter is designed to remove the trend from cycle of a series and therefore aid in diagnosing what is driving the data generation.
process for a series of data. Hodrick and Prescott define a time series of economic observations $y_t$ as consisting of a growth component $g_t$ and a cyclical component $c_t$:

$$y_t = g_t + c_t \text{ for } t = 1, \cdots, T$$

(3)

They assume that the smoothness of the growth path of $g_t$ over time can be measured by the sum of squares of its second difference. They further assume that over the long term that the cyclical components $c_t$ will sum to zero. With these assumptions the following programming problem can be solved for determining the growth components:

$$\min \left\{ \sum_{t=1}^{T} (y_t - g_t) + \lambda \sum_{t=1}^{T} [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2 \right\}$$

(4)

The parameter $\lambda$ is a coefficient that penalizes variability in the growth component series. Hodrick and Prescott suggest a value of 1,600 for $\lambda$ in quarterly data, implying a value of 400 in our annual data. However, Guay and St-Amant (2005) suggest analyzing the spectral density of the residuals from the filtering process for evidence that the filter removed high-frequency and low-frequency serial correlation from the data and then adjusting $\lambda$ as necessary to isolate the trend. In our case, values above 100 failed to remove the low-frequency serial correlation, so we ultimately chose a value of 50 for $\lambda$.

The third alternative model that we assessed was an ARIMA model estimated with a Kalman filter. ARIMA models are actually flexible modeling frameworks where the analyst chooses from a range of models with autoregressive parameters $\phi$ and moving average parameters $\theta$ and incorporating differencing as necessary to induce stationarity in the data.

The ARIMA model assumes that the data generation process for the time series conforms to the ARMA representation:

$$\phi(B)x_t = \theta(B)a_t$$

(5)
where the $x_t$ are the observations of revenue, $a_t$ are the residuals of the predictions of $x_t$, and $B$ is
the “backshift operator” that shows the number of lags of the autoregressive and moving average
process (indexed by $p$ for the autoregressive process and $q$ for the moving average, therefore an
ARMA ($p,q$) model with 1 lag of the autoregressive process and 2 moving average terms would
be referenced as an ARMA (1,2) model). The backshift introduces lagged values into the
analysis. Stationarity of the data is introduced through differencing the data as necessary to
remove trends. So an ARIMA (1,1,2) model would be the ARMA (1,2) model with the data first
differenced (the middle term in the ARIMA ($p,d,q$) represents the level of differencing). The
parameters of the ARIMA model are estimated through a linear filter, most often in modern
statistical software the filter used is the Kalman filter. The goal of an ARIMA approach is to find
the model that best fits the data through examining the various information criteria (AIC, BIC,
HQC) as well as examining residuals for serial correlation.

The results of the alternative estimations can be compared with the results of the trend
regression through an examination of the residuals from the estimates (Figures 3 and 4 for
taxable sales and property valuation respectively).

\[ x_t - \phi x_{t-1} = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} \]

\footnote{Written as a linear difference equation, an ARMA (1,2) process would be: $x_t - \phi x_{t-1} = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2}$}
Examining the results from Figures 3 and 4, for both sales and valuation the lowest standard deviation of residuals (the root mean squared error commonly used as a measure of forecast accuracy) is found in the Hodrick-Prescott filter, indicating it produces the most efficient
forecasts. In the taxable sales model, the Holt linear exponential smoothing estimator is the second most efficient while in the valuation model the ARIMA model was slightly more efficient. In all three models, the trend regression estimator was the least efficient, producing RMSEs nearly three times larger than the next least efficient estimator for taxable sales and 33 percent larger for total valuation.

Beyond the overall inefficiency of the trend regression estimator, it also produces estimates of the direction of the residual which are in contrast with those generated by more efficient models. Figure 5 shows the same time series plot of taxable sales and the prediction from the trend regression model as was shown in Figure 1. But in Figure 5 we overlay the predicted values from the Hodrick-Prescott filter. Not only is the fit of the prediction much better over time, there are several periods where the trend regression model would predict a positive deviation from trend while the Hodrick-Prescott filter would predict a negative deviation (for example, 1991-1996 and 2005-2006) and vice-versa (2007-2008).

These results are presented in a slightly different manner in Figure 6 for property valuation. The standardized residuals (residual as a percentage of actual value) of the Hodrick-Prescott filter are obviously smaller than those of the trend regression. And there are several instances (especially at the beginning and end of the sample period) where the signs of the residuals would be opposite.
Figure 5. Taxable Sales and Predictions from Time Trend Regression and Hodrick-Prescott Filter, 1975 - 2013.

Figure 6. Standardized Residuals from Trend Regression and Hodrick-Prescott Filter Estimation of Property Valuation, 1977-2013.
Conclusions

Based on our analysis, we conclude that the use of trend regression as a basis for generating measures of volatility produces estimates that are inflated compared to the actual volatility. This is because trend regression ignores the cyclical element of the time series data generation process. Previous research which has used measures of volatility generated in this manner need to be reevaluated in light of these findings. The results are especially problematic for research that has used volatility as a dependent variable and research that has used the sign of the residuals as a measure.

We have demonstrated three alternative methods for estimating the trend and cycle of the data and measuring the residual more efficiently. There are other methods that deserve mention and further analysis. First, autoregressive-conditional heteroscedastic models (ARCH) and generalized ARCH models (GARCH) are often used in the time-series modeling of financial data (see Engle 1982 and Bollerslev 1986). They produce estimates of the conditional variance of the data that is not accounted for by the trend, cycle, or seasonal components of the data. We attempted an ARCH estimate of the data, but the model did not indicate that ARCH was appropriate (the conditional variance terms were insignificant). This is not surprising with annual data as volatility-clustering ARCH processes tend to take place over relatively short periods of time. The second class of methods to mention are unobserved component models. These models include ARIMA models and classical decomposition models as special cases, but go far beyond them. They provide an extremely flexible way of modeling time series to include changes in the level of the series over time (“local levels models”) and changes in exogenous variables, and produce distinct estimates for the trend, cyclical, seasonal, and residual elements of a series (Harvey 1989).
Whatever method is ultimately decided, we forward that the emphasis in measuring revenue volatility should shift to those models that produce the best estimates of the trend, cycle, and seasonal components of a time series and therefore produce the cleanest measures of volatility. We look forward to an ongoing discussion of this topic.

References


