Abstract:
In 1957 Hirschman proved that the sum of entropies of a function $f$ (with $\|f\|_2 = 1$) and its Fourier transform is nonnegative. He also observed that a stronger version of this inequality:

$$
- \int_{\mathbb{R}} |f(x)|^2 \log(|f(x)|^2) dx - \int_{\mathbb{R}} |\hat{f}(\xi)|^2 \log(|\hat{f}(\xi)|^2) d\xi \geq \log\left(\frac{e}{2}\right)
$$

(proven later by Beckner) implies the Heisenberg Uncertainty Principle. Hirschman conjectured that the minimizers for the sharp inequality (1) were Gaussians, as is the case for the Heisenberg Uncertainty Principle. We have shown that this is indeed the case.

There is an analog of (1) for functions $f$ defined on a finite abelian group, with applications in Signal Processing. The shall also describe these minimizers. They depend on the structure of the finite abelian group and are not “Gaussians” or discretized Gaussians. This discrepancy between the finite and the continuous case seems to be unexpected in the Signal Processing community.

Friday, September 29, 2000
3:00 PM in 335 Jabara Hall

Please come join us for refreshments before the lecture
at 2:30 p.m. in room 353 Jabara Hall.