"The character of initial-boundary value problems and their numerical solution"

Abstract:
It is well known that solutions of elliptic partial differential equations (PDEs) are usually singular (not smooth) in the corners of the spatial domain. These ‘corner singularities’ have observable consequences in science and engineering; in buildings, cracks radiate from the corners of walls, windows and doors; airplanes have rounded windows to minimize cracks that can cause catastrophic failures; small nested vortices called “Moffatt eddies” appear in the corners of fluid-filled regions. What has gone essentially unnoticed in the numerical literature is that solutions of virtually all initial-boundary value problems will exhibit singularities in the corners of the time-space domain, unless the initial and boundary data satisfy an infinite set of compatibility conditions. Since these two sets of data almost always arise from independent considerations, these conditions are very rarely satisfied. The nature of these singularities is elusive and their impact on numerical calculations is severe. Without special treatment, the accuracy of high-order methods will be reduced to that of a low-order scheme. Analytical and numerical tools to analyze and remedy these time-space corner singularities will be discussed in the cases of diffusive and dispersive PDEs. In particular, this includes novel contour integral representations of the solutions in the complex plane by Professor Fokas and our efficient numerical evaluation of them.

Tuesday, August 28, 2007
3:00 PM in 353 Jabara Hall