ME 650P
CFD for Thermal/Fluid Analysis and Design
Midterm Exam
November 25, 2002

Instructions

1. This is a closed-book/closed-notes examination.

2. Please ask the exam proctor if you need clarification with any of the question statements. If you need any additional equations/formulae, please ask.

3. Provide concise answers in the space provided below each question, but feel free to use additional paper if necessary. Write your name down on every sheet of paper that you intend to be graded.

Name:

Grades:

1.
2.
3.
4.
5.
6.

Total:
Some useful definitions and formulae

- Derivatives:
  \[ \nabla(\star) \equiv i \frac{\partial(\star)}{\partial x} + j \frac{\partial(\star)}{\partial y} + k \frac{\partial(\star)}{\partial z}; \quad \nabla^2(\star) \equiv \frac{\partial^2(\star)}{\partial x^2} + \frac{\partial^2(\star)}{\partial y^2} + \frac{\partial^2(\star)}{\partial z^2} \]
  \[ \frac{D(\star)}{Dt} \equiv \frac{\partial(\star)}{\partial t} + \nabla \cdot (\star) \nabla(\star) \]

- The Continuity Equation:
  \[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \]

- The viscous stress tensors:
  \[ \tau_{xx} = \lambda(\nabla \cdot \mathbf{V}) + 2\mu \frac{\partial u}{\partial x}; \quad \tau_{yy} = \lambda(\nabla \cdot \mathbf{V}) + 2\mu \frac{\partial v}{\partial y}; \quad \tau_{zz} = \lambda(\nabla \cdot \mathbf{V}) + 2\mu \frac{\partial w}{\partial z} \]
  \[ \tau_{xy} = \tau_{yx} = \mu \left[ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right]; \quad \tau_{yz} = \tau_{zy} = \mu \left[ \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right]; \quad \tau_{zx} = \tau_{xz} = \mu \left[ \frac{\partial w}{\partial z} + \frac{\partial w}{\partial x} \right] \]

In the above, \( \mu \) and \( \lambda \) are the first and the second coefficients of viscosity, respectively. For a \textit{newtonian} fluid, \( \mu \) is a constant, and Stokes’ hypothesis holds, i.e., \( \lambda = -\frac{2}{3}\mu \).

- The Navier-Stokes Equations:
  \[ \rho \frac{Du}{Dt} = \rho g_x - \frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \]
  \[ \rho \frac{Dv}{Dt} = \rho g_y - \frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \]
  \[ \rho \frac{Dw}{Dt} = \rho g_z - \frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \]

- The Energy Equation (conservation of thermal energy):
  \[ \rho c_p \frac{DT}{Dt} = k \nabla^2 T + \rho q + \rho \phi \]

where \( \phi \) expresses all deformation work (per unit mass) resulting in heating, or \textit{dissipation} of mechanical energy into \textit{thermal} form.
Question 1.a (for graduate students only)
Show that the average of the three normal viscous stresses is zero, i.e.:

\[
\frac{1}{3} \left[ \tau_{xx} + \tau_{yy} + \tau_{zz} \right] = 0
\]

at any point in a fluid flow where Stokes’ hypothesis is valid.

Question 1.b (for undergraduate students only)
Explain the physical significance of each of the terms in the “continuity equation”. What phenomena does this equation represent?
Question 2 (answer only one of the following)

(a) Explain the physical significance of each of the terms in one of the three components of the Navier-Stokes equations. What phenomena does this equation represent?

(b) What does the following expression signify? Explain the physical significance of each of the terms.

\[ \frac{D(\ast)}{Dt} = \frac{\partial(\ast)}{\partial t} + \mathbf{V} \cdot \nabla(\ast) \]
Question 3

As explained in class, second order partial differential equations (PDEs) can be classified into three types, each describing a different kind of physical problem. Pair each of the following heat transfer/fluid flow situations with the type of PDE that describes (or "governs") the problem:

(a) Supersonic flow of air and heat transfer over a missile head   (i) Elliptic PDE
(b) Subsonic flow and development of a boundary layer over a flat plate   (ii) Hyperbolic PDE
(c) Natural convection inside a climate controlled room   (iii) Parabolic PDE

Question 4

What are the sources of numerical errors in any computational scheme used in the modeling of physical phenomena? How can you eliminate/minimize these sources?
Question 5

Briefly describe how you can ensure your computational predictions of a fluid flow or heat transfer problem are reasonably correct and therefore representative of a real situation.
Question 6
Which numerical method is FLUENT based upon: (i) Finite Difference; (ii) Finite Volume; (iii) Finite Element? What may be the possible limitations/advantages of FLUENT as compared to other similar software packages?

Question 7
Briefly explain any three of the following terminology: (a) Stability; (b) Consistency; (c) Convergence; (d) "iterative convergence"; (e) "well posed" problems.
Question 8

What are the most basic issues to be considered in designing a thermal system? Briefly explain (or illustrate with examples) each of these issues.

Show the various major steps in a design process using a flow diagram. Where does CFD research fit into this scheme? (Hint: CFD activities may fit into more than one location in such a scheme.)

List some of the issues specific to design and analysis of thermal/fluid systems.