Orbits of Complex Numbers Lab

Finding the orbit of \( x_0 = 0 + 0i \) in an equation with complex coefficients is somewhat more difficult than when using an equation with real coefficients. The process is exactly the same, it's just that using complex numbers adds another level of difficulty.

**EXAMPLE 1:**
Find the first five terms of the orbit of \( x_0 = 0 + 0i \) in the following equation:
\[
f(x) = x^2 + (0 + i)
\]

\[
f^1(x) = (0 + 0i)^2 + (0 + i) = 0 + i
\]
\[
f^2(x) = (0 + i)^2 + (0 + i) = -1 + i
\]
\[
f^3(x) = (-1 + i)^2 + (0 + i) = 0 - i
\]
\[
f^4(x) = (0 - i)^2 + (0 + i) = -1 + i
\]
\[
f^5(x) = (-1 + i)^2 + (0 + i) = 0 - i
\]

a. Does there appear to be a pattern developing? What is it?

b. \( f^{10}(x) = ? \)
c. \( f^{101}(x) = ? \)

d. Is the orbit of \( x_0 \) in the equation \( f(x) = x^2 + (0 + i) \) periodic? What is the period?

**When does an orbit escape to infinity?**

The orbit of \( x_0 \) in the equation \( f(x) = x^2 + c \), where \( c = a + bi \), escapes to infinity if \( |f^n(x_0)| > 2 \) for some value of \( n \). Note: \( |c| = \sqrt{a^2 + b^2} \)

**EXAMPLE 2:**
Find the first five terms of the orbit of \( x_0 = 0 + 0i \) in the following equation:
\[
f(x) = x^2 + (1 + i)
\]

\[
f^1(x) = (0 + 0i)^2 + (1 + i) =
\]
\[
f^2(x) = (1 + i)^2 + (1 + i) =
\]
\[
f^3(x) = (1 + 3i)^2 + (1 + i) =
\]
\[
f^4(x) = (-7 + 7i)^2 + (1 + i) =
\]
\[
f^5(x) = (1 - 97i)^2 + (1 + i) =
\]

a. Is the orbit of \( x_0 \) escaping to infinity? At what term is this clear?

b. Let \( f(x) = x^3 + (1 + bi) \). What values could we put in for "\( b \)" and be sure the orbit of
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1. Let \( x_o = 0 + 0i \) in the equation \( f(x) = x^2 + (-0.5 + 0.5i) \).
   a. Find the first five terms of the orbit of \( x_o = 0 + 0i \).
   b. Graph the answers to part a) on the complex plane at the right.
   c. Use the program \textbf{CXORBZOM} or \textbf{CXORBLST} and look at a more complete picture of the orbit of \( x_o \). Make a sketch of it below. What appears to be happening?

2. Let \( x_o = 0 + 0i \) in the equation \( f(x) = x^2 + (-0.75 + 0i) \).
   a. Find the first five terms of the orbit of \( x_o = 0 + 0i \).
   b. Graph the answers to part a) on the complex plane at the right.
   c. Use the program \textbf{CXORBZOM} or \textbf{CXORBLST} and look at a more complete picture of the orbit of \( x_o \). Make a sketch of it below. What appears to be happening?
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3. Let \( x_0 = 0 + 0i \) in the equation \( f(x) = x^2 + (0 + 0.5i) \).

a. Find the first five terms of the orbit of \( x_0 = 0 + 0i \).

b. Graph the answers to part a) on the complex plane at the right.

c. Use the program \texttt{CXORBZOM} or \texttt{CXORBLST} and look at a more complete picture of the orbit of \( x_0 \). Make a sketch of it below. What appears to be happening?

4. Let \( x_0 = 0 + 0i \) in the equation \( f(x) = x^2 + (0.6 + 0.8i) \).

a. Find the first five terms of the orbit of \( x_0 = 0 + 0i \).

b. Graph the answers to part a) on the complex plane at the right.

c. Use the program \texttt{CXORBZOM} or \texttt{CXORBLST} and look at a more complete picture of the orbit of \( x_0 \). Make a sketch of it below. What appears to be happening?
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5. Let $x_0 = 0 + 0i$ in the equation $f(x) = x^2 + (-0.5 + 0.6i)$.
   
   a. Find the first five terms of the orbit of $x_0 = 0 + 0i$.

   b. Graph the answers to part a) on the complex plane at the right.

   c. Use the program CXORBZOM or CXORBLST and look at a more complete picture of the orbit of $x_0$. Make a sketch of it below. What appears to be happening?

6. Let $x_0 = 0 + 0i$ in the equation $f(x) = x^2 + (0 + 0.45i)$ and in $f(x) = x^2 + (0 - 0.45i)$.

   a. Find the first five terms of the orbit of $x_0 = 0 + 0i$.

   b. Graph the answers to part a) on the complex plane at the right.

   c. Use the program CXORBZOM or CXORBLST and look at a more complete picture of the orbit of $x_0$ in each equation. Make a sketch of the orbits below. How do the two graphs compare?