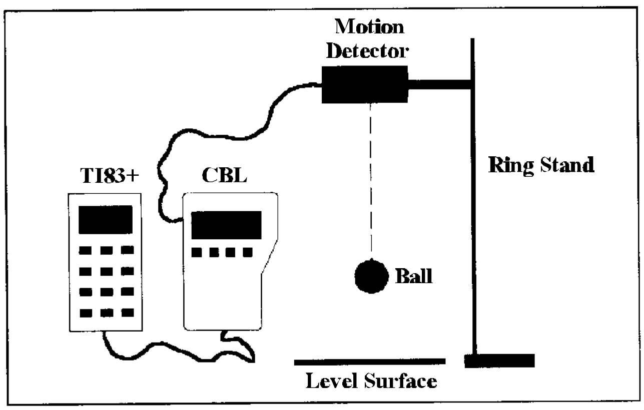
Name ____________________ Hour _____

Analyzing a Falling Object Lab

A function $s(t)$ that gives the position of an object as a function of time t is a **position function**. The position of a free-falling object (neglecting air resistance) under the influence of gravity can be modeled by the following equation:

$$s(t) = -\frac{1}{2}gt^2 + v_0 t + h_0$$

where g is the acceleration of gravity, t is time, v_0 is the initial velocity, and h_0 is the initial height



In this lab, you will gather position and time data on a falling object using a CBL and motion detector. You will then use this data to develop a mathematical model for the position of the object at any time t.

1. Set up the lab as instructed by your teacher.

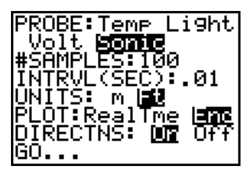
2. You will be using the **CBL/CBR** program on your calculator to gather the data. Follow the instructions below to set up the program and its parameters for this lab.

Procedure To Operate *CBL/CBR* Program On The TI83+ Calculator:

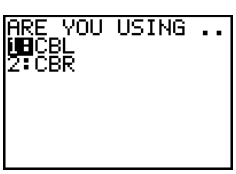
Press the **APPS** key and select **2** for **CBL/CBR**.

Press **ENTER**.

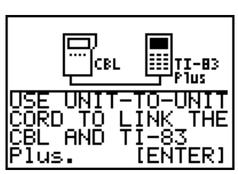
Select **2** for **DATA LOGGER**.



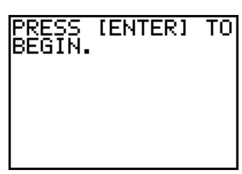
Set the parameters as shown. Turn on the directions the first time. Arrow to **GO** and press **ENTER**.



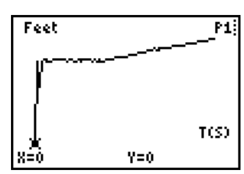
Select **1** for **CBL**.



There will be a number of screens checking various connections. Keep pressing **ENTER**.

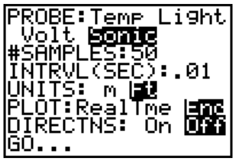


At this screen, when you press **ENTER**, the **CBL** will start collecting data after a short delay.

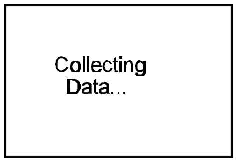


If you are unhappy with your graph, press **ENTER** and start over.

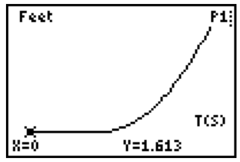
Analyzing a Falling Object Lab



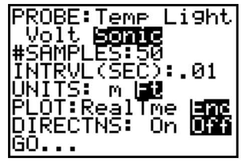
Turn off the directions to save time. Arrow down to **GO**. When you press **ENTER**, the **CBL** will start after a short delay.



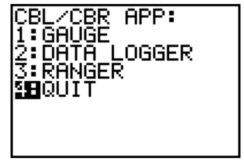
This screen indicates that data is being collected by the **CBL**.



When you are satisfied with your graph, press **ENTER**.



At this screen, press **2nd MODE** for **QUIT**.



Press **4** to **QUIT** the program.

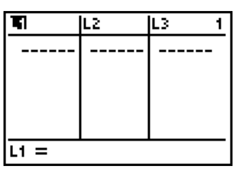
3. Every member of your group must transfer the data, stored in lists **TDIST** (sec) and **DIST** (ft), into their calculator.

4. Save your data lists to a program. Your Program Name: ____________________

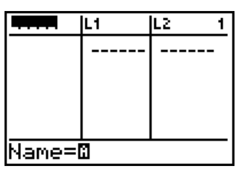
After you have collected the data and are satisfied with your graph, there will probably be parts of the graph that you do not want to keep, especially at the beginning and the end of the graph. To **select** out only that part of the graph you wish to keep, follow the instructions below.

Entering The Lists TDIST And DIST Into The LIST Screen:

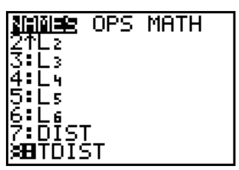
1. First, please note that your data is being stored in the lists **TDIST** (time-sec) and **DIST** (distance-ft). To show them in your **LIST** screen, follow the directions below.



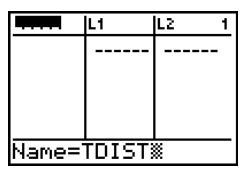
Go to the top of L_1 and press **2nd DEL** for **INS**.



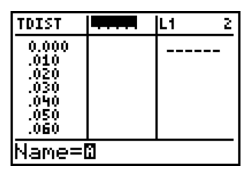
You have the option of typing in **TDIST** or accessing it under the list names. See next screen for this.



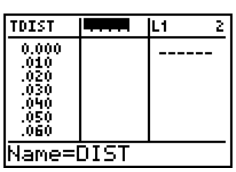
Press **2nd STAT** for **LIST** and select **8** for the list **TDIST**.



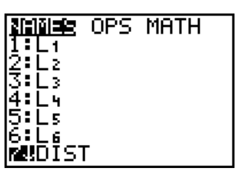
Press **ENTER** and the data for **TDIST** will be entered.



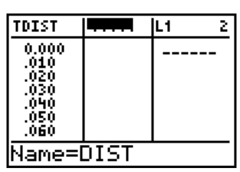
Go back to the top of L_1 and press **2nd DEL** for **INS**.



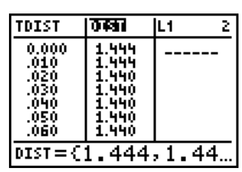
Either type in **DIST** or see next screen to select it from the list names.



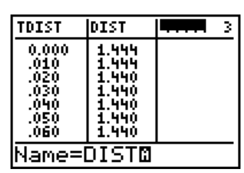
Press **2nd STAT** for **LIST** and select **7** for the list **DIST**.



Press **ENTER** and the data for **DIST** will be entered.



You now have the data available at your **LIST** screen.

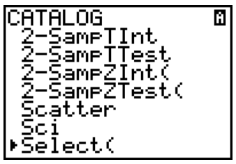


Insert another list at the top of L_1 and type in **DIST**. What will happen when you press **ENTER**?

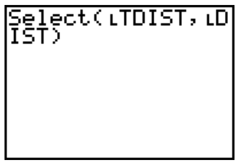
Analyzing a Falling Object Lab

Selecting Out Part Of A Graph:

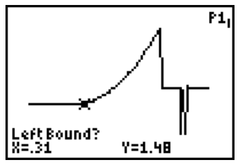
2. You are now ready to **select** out the part of the graph you wish to keep. Follow the instructions below to accomplish this.



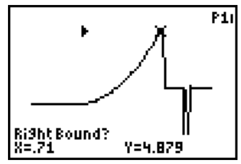
Press **2nd 0** for **CATALOG** and the letter **S** to go to the **S**-screen. Arrow down to **Select(** and press **ENTER**.



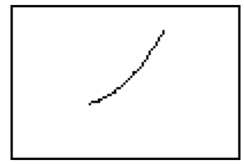
At the home screen, enter **TDIST**, **DIST**. Note: These **cannot** just be typed in. (See the little ʟ in front of the names?) Access them at the **LIST** screen and then press **ENTER**.



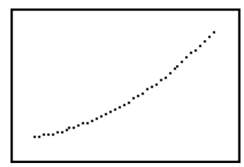
Cursor to the left bound of the part you want to keep and then press **ENTER**.



Cursor to the right bound of the part you want to keep and press **ENTER**.



To maximize the view of the graph, using **Dot** mode, press **ZoomStat**.



Sketch the graph of the (modified) time vs. position data in the space at the right.

1. This selection process automatically stores the modified data back into the original data lists, deleting it out completely. If we need the original data, we can access the program in which it is stored. If possible, get a printout of your graph from your instructor.

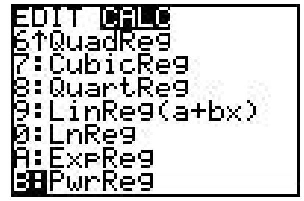
2. What parent functions would make reasonable models for this graph?

3. Identify the common name of each of the following functions and sketch a representative graph of each.

 a. $y = ax^2 + bx + c$ b. $y = ax^3 + bx^2 + cx + d$

 c. $y = bx^n$ d. $y = ab^x$

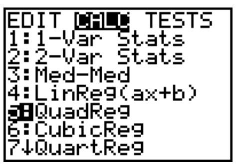
Analyzing a Falling Object Lab



To help find a function that best models your (modified) position vs. time data, go to the **STAT** feature on your calculator and arrow to **CALC**. Here you will find a number of equations the calculator will fit to your data. The process of fitting an equation to a set of data is called *regression analysis*. Follow the directions below to find a good model for your data.

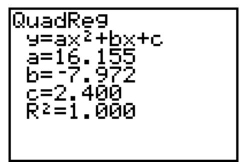
CAUTION: The data only allows us to look at a relatively small part of the whole curve. Before you select a function as the best fitting model, always ask yourself if the graph of the whole equation reflects the conditions of the situation.

1. Use **QuadReg** to fit a quadratic equation to your (modified) position vs. time data. QuadReg stands for "quadratic regression" and refers to a process that finds the best fitting quadratic equation to the data.



At the home screen, press **STAT**, arrow to **CALC**, and press **5**.

Enter **TDIST, DIST,** $\mathbf{Y_1}$ and then **ENTER**.



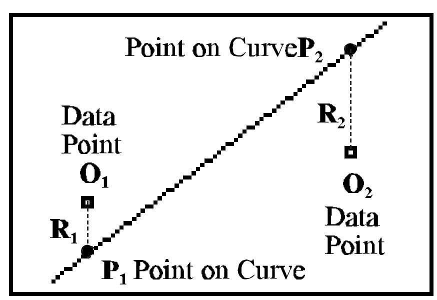
Your screen should look like this.

a. Write the equation found in $\mathbf{Y_1}$. Round all numbers to the nearest thousandth.

Equation 1: $y =$

b. Sketch **Equation 1** and the (modified) position vs. time data on the same axes in the space at the right. Comment on the fit of **Equation 1** to the data.

c. Each time the calculator runs a regression analysis, it generates a list of *residuals* that it stores in the list **RESID** (found in **LIST** names). A residual represents the distance between the actual (or observed) *y*-value of a data point and the predicted *y*-value (using the regression equation). The smaller the residuals, the better the equation fits the data.



The residual $\mathbf{R_1} = \mathbf{O_1} - \mathbf{P_1}$ and is positive, since $\mathbf{O_1} > \mathbf{P_1}$.
The residual $\mathbf{R_2} = \mathbf{O_2} - \mathbf{P_2}$ and is negative, since $\mathbf{O_2} < \mathbf{P_2}$.

Analyzing a Falling Object Lab

1) Graph the residuals in the space at the right.

2) Look at the window values and comment on the relative size of the residuals compared to the actual data's values.

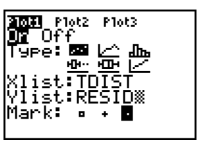
2. Use **CubicReg** to fit a cubic equation to your data. CubicReg stands for "cubic regression" and refers to a process that finds the best fitting cubic equation to the data.

   a. Use **CubicReg** to fit a cubic equation to your (modified) position vs. time data.

   b. Write the equation found in $\mathbf{Y_1}$. Round all numbers to the nearest thousandth.

   Equation 2: $y =$

   c. Sketch **Equation 2** and the (modified) position vs. time data on the same axes in the space at the right. Comment on the fit of **Equation 2** to the data.



1) Graph the residuals in the space at the right.

2) Look at the window values and comment on the relative size of the residuals compared to the actual data's values.

3. Use **PwrReg** to fit a power equation to your data. PwrReg stands for "power regression" and refers to a process that finds the best fitting power equation to the data. **NOTE:** If you have a value of zero for one of your *x*-values (time), you will need to change it to 0.0001 before **PwrReg** will work.

   a. Use **PwrReg** to fit a power to your (modified) position vs. time data.

   b. Write the equation found in $\mathbf{Y_1}$. Round all numbers to the nearest thousandth.

   Equation 3: $y =$

c. Sketch **Equation 3** and the (modified) position vs. time data on the same axes in the space at the right. Comment on the fit of **Equation 3** to the data.

1) Graph the residuals in the space at the right.

2) Look at the window values and comment on the relative size of the residuals compared to the actual data's values.

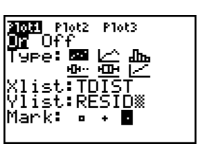
4. Use **ExpReg** to fit an exponential equation to your (modified) position vs. time data. ExpReg stands for "exponential regression" and refers to a process that finds the best fitting exponential equation to the data. **NOTE:** If you have a value of zero for either your *x*- or *y*-value, you will need to replace the zero with 0.0001 before **ExpReg** will work.

a. Use **ExpReg** to fit an exponential equation to your (modified) position vs. time data.

b. Write the equation found in $\mathbf{Y_1}$. Round all numbers to the nearest thousandth.

Equation 4: $y =$

c. Sketch **Equation 4** and the (modified) position vs. time data on the same axes in the space at the right. Comment on the fit of **Equation 4** to the data.



1) Graph the residuals in the space at the right.

2) Look at the window values and comment on the relative size of the residuals compared to the actual data's values.

5. After examining each of the regression analysis, which of the equations best models the (modified) time vs. position data? Explain.

Of course, the theoretical model for a freely falling body (neglecting air resistance) under the influence of gravity is a quadratic equation. We have discussed this at some length in class.

1. Write out the **theoretical position equation** for a freely falling body.

   $s_T(t) =$

2. Rewrite **Equation 1** below to reflect the fact that it is the **experimental position equation**.

   $s_E(t) =$

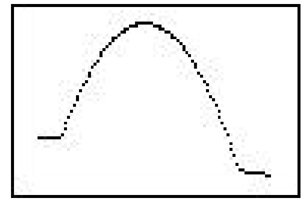
3. Discuss the relationship between the coefficients of $s_E(t)$ and $s_T(t)$ and what these coefficients represent physically.

4. How might you explain any differences between the experimental values and the theoretical values of the coefficients?

5. Use the space at right to show the **complete** graph of $s_E(t)$, the experimental position equation.

Analyzing a Falling Object Lab

In a related experiment, a group of students got the results at right. They found that a quadratic equation was the best model for the (modified) time vs. position data. The vertical axis is position and the horizontal axis is time. The origin is at the lower left of the screen. Note: The diagram shows the graph before the group selected out the part they kept.



1. Describe what the design of their experiment might have been to get this data.

2. Indicate on the graph where the object reached its maximum height. What's the mathematical term for this maximum point?

3. What is the translation that would move the point (0, 0) to (3, 5)? Use proper mathematical notation.

4. Which of the following equations would best represent the data (excluding the ends), under the translation $T_{3,5}$, if the maximum height reached was 5 meters. _____

   a. $y = -2(x - 3)^2 - 5$ b. $y = -(x + 3)^2 + 5$ c. $y = -0.5(x - 3)^2 + 5$

5. Consider the ratio $\frac{s_E(t_1) - s_E(t_2)}{t_1 - t_2} = \frac{\Delta s}{\Delta t}$ (Read as delta s over delta t). This ratio represents the rate of change of position with respect to the change in time.

   a. What is a more common name for this ratio? Hint: Look at the units in the numerator and denominator that we used in the lab.

   b. Is the velocity of the falling object increasing at an increasing rate, increasing at a decreasing rate, increasing at a constant rate, decreasing at a constant rate, decreasing at an increasing rate, or a decreasing at a decreasing rate?

Analyzing a Falling Object Lab

For The Calculus Students:

We have discussed the relationship between position, velocity, and acceleration and will address these relationships throughout the course. In our discussions, we have differentiated between **average** velocity and acceleration (over some time interval Δt) and **instantaneous** velocity and acceleration (at an instant in time).

1. Assume that $s(t)$ is a position function.

    a. Write out an equation for the average velocity over some time interval Δt .

    b. Write out what the instantaneous velocity would be at an instant in time using the definition of derivative.

Experimental Instantaneous Velocity ($v_E(t)$) vs. Theoretical Instantaneous Velocity ($v_T(t)$):

1. What is the relationship between $v_E(t)$ and $s_E(t)$?

2. Use $s_E(t)$ to find the experimental instantaneous velocity function for the falling object.

    $v_E(t) =$

3. Use $s_T(t)$ to find the theoretical instantaneous velocity function for the falling object.

    $v_T(t) =$

4. Compare $v_E(t)$ and $v_T(t)$. How would you account for any differences?

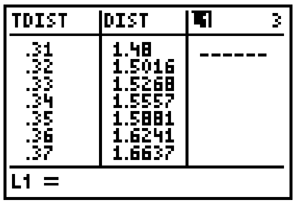
Analyzing a Falling Object Lab

5. By following the procedure below, we are going to see how good a model we can find for $v_E(t)$ using our (modified) position vs. time data and the concept of average velocity., i.e.

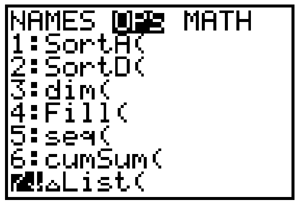
$$\text{Average Velocity}(\Delta v) = \frac{\Delta s}{\Delta t} = \frac{\text{Change In Position}}{\text{Change In Time}}$$

a. In this lab, what would $\mathbf{DIST}(n+1) - \mathbf{DIST}(n)$ represent?

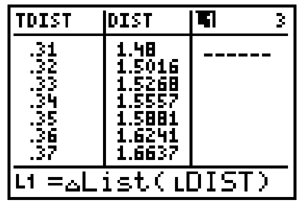
Procedure To Find Change In Position:



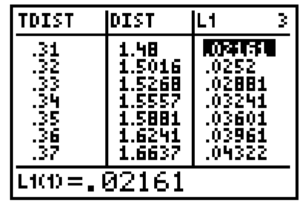
Go to the top of $\mathbf{L_1}$.



Press **2nd STAT** for **LIST**, arrow to **OPS**, and select **7** for **ΔLIST**.



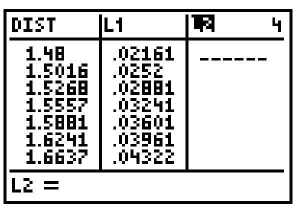
Your screen should look like this. Press **ENTER**.

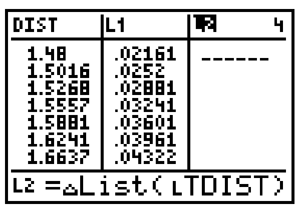


What do the values in $\mathbf{L_1}$ represent?

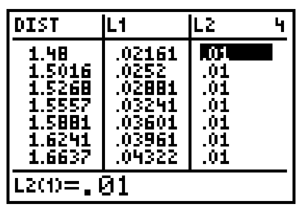
b. In this lab, what is Δt ?

Procedure To Find Δt :





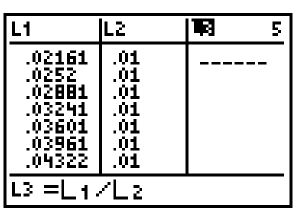
What is this going to do?

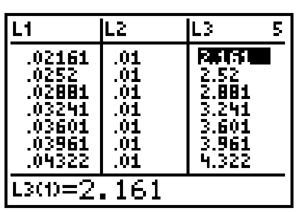


Surprise?

c. We are now going to make a list representing the experimental average velocities.

Procedure To Find Experimental Average Velocities:





1) What do the values in $\mathbf{L_3}$ represent?

2) How many values are in $\mathbf{L_3}$?

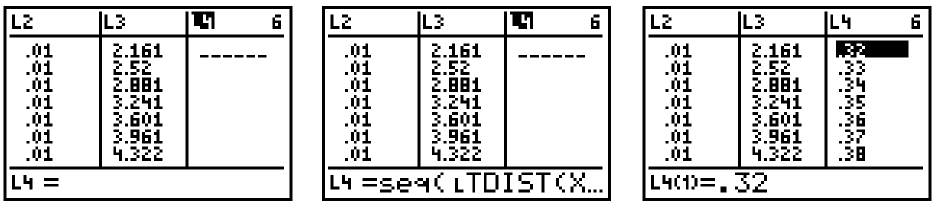
3) What's the relationship

between $\frac{\mathbf{L_1}}{\mathbf{L_2}}$ and $\frac{\Delta s}{\Delta t}$?

Analyzing a Falling Object Lab

d. The last step before finding a model for $v_E(t)$ is to find the time at which each average velocity occurred.

Procedure For Finding The Time At Which Each Average Velocity Occurred:



Seq(LTDIST(X),X,2,40,1)

1) What do the values in $\mathbf{L_4}$ represent?

2) How many values are in $\mathbf{L_4}$?

e. Finally, we are in a position to find a model for $v_E(t)$ based on our data.

1) Sketch a graph of ($\mathbf{L_4}$, $\mathbf{L_3}$) using **STATPLOT** (**Dot** mode) in the space at the right. What type of function does the graph appear to be?

2) Perform a **LinReg** on the data in $\mathbf{L_4}$ and $\mathbf{L_3}$ and write the resulting linear equation below.

Equation 5: $y =$

6. Compare **Equation 5**, $v_E(t)$: the experimental instantaneous velocity equation from the derivative of our experimental position function, and $v_T(t)$: the theoretical instantaneous velocity equation from the derivative of the theoretical position function. Include a sketch of all three equations on the same axes in the space at the right.

Analyzing a Falling Object Lab

Experimental Instantaneous Acceleration ($a_E(t)$) vs. Theoretical Instantaneous Acceleration ($a_T(t)$):

1. What is the relationship between $a_E(t)$ and $v_E(t)$?

2. Use $v_E(t)$ to find the experimental instantaneous acceleration function for the falling object.

   $a_E(t) =$

3. Use $v_T(t)$ to find the theoretical instantaneous velocity function for the falling object.

   $a_T(t) =$

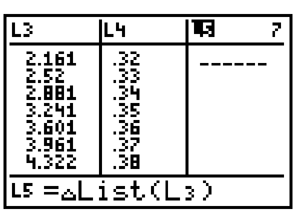
4. Compare $a_E(t)$ and $a_T(t)$. How would you account for any differences?

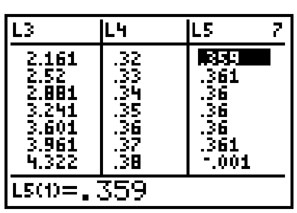
5. By following the procedure below, we are going to see how good a model we can find for $a_E(t)$ using our (modified) position vs. time data and the concept of average acceleration.

   $$\text{Average Acceleration} = \frac{\Delta v}{\Delta t} = \frac{\text{Change In Velocity}}{\text{Change In Time}}$$

   a. In this lab, what is Δt ?

   b. We will need to create a list representing the change in velocity. Do this in $\mathbf{L_5}$.





Procedure For Finding The Change In Velocity:

1) What do the values in $\mathbf{L_3}$ represent?

2) What do the values in $\mathbf{L_5}$ represent?

Analyzing a Falling Object Lab

c. Since Δt hasn't changed, we could again use $\mathbf{L_2}$ to represent Δt . However, we would need to pay attention to the number of values in $\mathbf{L_2}$ and the number of values in $\mathbf{L_5}$. What could you do to $\mathbf{L_2}$ so that it has the same number of values in it as does $\mathbf{L_5}$?

d. We are now in a position to make a list of the experimental average accelerations.

Procedure To Find Experimental Average Accelerations:

L4	L5	L6 8
.32	.359	------
.33	.361	
.34	.36	
.35	.36	
.36	.36	
.37	.361	
.38	⁻.001	

L6 =L5/.01

L4	L5	L6 8
.32	.359	35.9
.33	.361	36.1
.34	.36	36
.35	.36	36
.36	.36	36
.37	.361	36.1
.38	⁻.001	⁻.1

L6(1)=35.9

1) What do the values in $\mathbf{L_5}$ represent?

2) What do the values in $\mathbf{L_6}$ represent?

e. The last step before finding a model for $a_E(t)$ is to find the time at which each average acceleration occurred. This is done by using the same technique you did for average velocity. Can we just go ahead and use the times we found for the average velocities?

f. Finally, we are in a position to find a model for $a_E(t)$ based on our data.

1) Sketch a graph of ($\mathbf{L_4}$, $\mathbf{L_6}$) using **STATPLOT** (**Dot** mode) in the space at the right. What type of function does the graph appear to be?

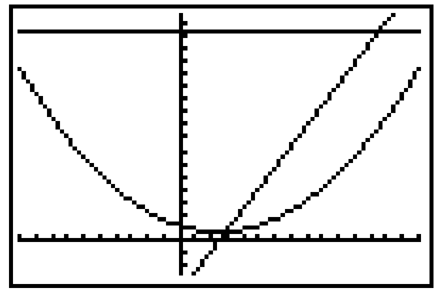
2) Perform a **LinReg** on the data in $\mathbf{L_4}$ and $\mathbf{L_6}$ and write the resulting linear equation below.

Equation 6: $y =$

6. Compare **Equation 6**, $a_E(t)$: the experimental instantaneous acceleration equation from the derivative of our experimental velocity function, and $a_T(t)$: the theoretical instantaneous acceleration equation from the derivative of the theoretical velocity function. Include a sketch of all three equations on the same axes in the space at the right.

Analyzing a Falling Object Lab

It is very important that you understand the relationships between $s_T(t)$, $v_T(t)$, and $a_T(t)$, both algebraically and geometrically. Check your understanding by answering the following questions.



1. Consider the diagram shown at the right of the graphs of $s_T(t)$, $v_T(t)$, and $a_T(t)$.
   a. Identify which graph represents the functions $s_T(t)$, $v_T(t)$, and $a_T(t)$.

   b. Using **only the graph**, how could you verify where the relative minimum $s_T(t)$ of occurs?

   c. Using **only the graph**, how could you verify where $s_T(t)$ is increasing? Decreasing?

   d. Using **only the graph**, how could you verify that $s_T(t)$ is concave up?

   e. Using **only the graph**, how could you verify that $v_T(t)$ is increasing?

2. What is the relationship between $a_T(t)$ and $v_T(t)$? between $a_T(t)$ and $s_T(t)$? between $v_T(t)$ and $s_T(t)$?

3. What is the relationship between $v_T(t)$, $s_T(t)$, Δs, and Δt?

4. What is the relationship between $a_T(t)$, $v_T(t)$, Δv, and Δt?

5. Assume that $s_T(t)$ was entered in $\mathbf{Y_1}$. Describe how you could graph $v_T(t)$ and $a_T(t)$ using the **nDeriv** feature of your calculator.

6. Summarize the relationships between $s_T(t)$, $v_T(t)$, and $a_T(t)$.