Name ____________________ Hour _____

Exploring the Number "e" Lab

Part 1 - Finding a Maximum Value

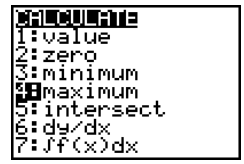
1. In the space at right, graph the function $f(x) = x^{\frac{1}{x}}$.

[0, 5] x [0, 2]

a. Use the trace feature of your calculator to get an estimate of the maximum point (3 decimal places).

Estimated maximum point (,).

b. Now, use the **CALCULATE** feature of your calculator to find the maximum point. Go to **2nd TRACE** for **CALCULATE** and select **4** for **maximum**. Follow the directions on the screen. Be sure you're to the **left of the max** (lower bound) and then to the **right of the max** (upper bound).



Actual maximum point (,) to the billionths place.

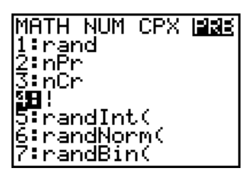
c. Make a conjecture as to the **exact value** of the *x*-value and *y*-value of the maximum point.

Part 2 - Another Name for the Function e^x

2. In 1748, **Leonhard Euler** (pronounced "oiler"), published a work in which he developed an irrational number that ranks along with π in importance. In his honor the number is called ***e***, the **Euler number**. One way that ***e*** can be found is by finding the "limit" of the following infinite series.

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \cdots + \frac{1}{(n-1)!} + \cdots$$

Note: As an example, 4! = 4x3x2x1.



To access "!," the factorial symbol, press **MATH**, arrow to **PRB**, and select **4**.

a. Write out the first six terms of the series.

b. Is the series arithmetic or geometric? Explain

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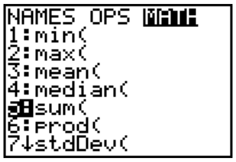
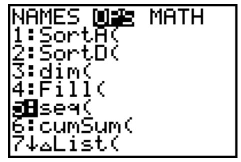
c. Evaluate the expression for the number of terms given in the table below. To do this, go to the home screen of your graphics calculator and do the following:



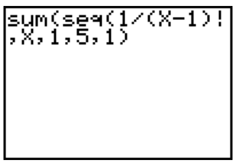
Press **2nd STAT** for **LIST**. Arrow to **MATH** and select **5** for **sum** and press **ENTER**.

Depending on your calculator, you should see this on your home screen.

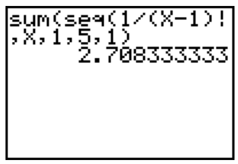


Press **2nd STAT** for **LIST**. Arrow to **OPS** and select **5** for **seq** and press **ENTER**.

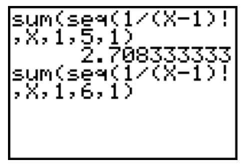
Depending on your calculator, you should see this on your home screen.



Type in the above information to find the sum of the first 5 terms of the series.



When you press **ENTER**, the sum of the first 5 terms is shown.



Press **2nd ENTER** to bring back your last entry. Arrow to the 5 and change it as needed.

Number of terms	**Sum of the series** (write down all decimal places shown)		Number of terms	**Sum of the series** (write down all decimal places shown)
5			9	
6			10	
7			15	
8			20	

d. What's the **minimum** number of terms that must be added so that the sum of the series represents ***e*** to nine (9) decimal places?

e. Does the limit of the series seem to "converge" to its limiting value, ***e***, quickly?

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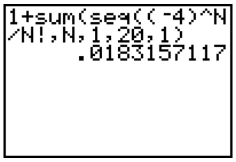
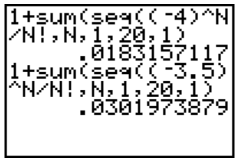
Part 3 - Using the Slope Feature

3. Consider the following function $f(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^{n-1}}{(n-1)!} + \cdots$.

a. Fill in the table below by evaluating $f(x)$ at the x-values shown in the table.





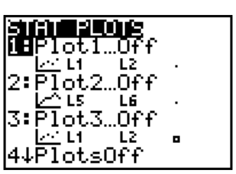
Follow the procedure shown at the left. Use the **2nd ENTER** feature to bring back the last entry and edit it for the next x-value.

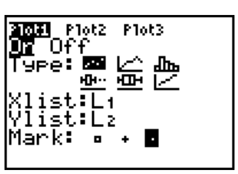
(1) Why is "N" used rather than "N-1" for the expression shown on the screen?

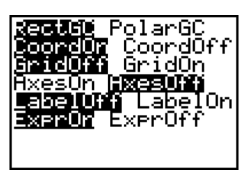
(2) How many terms are you adding in the procedure to get an approximation for $f(x)$?

Value of x	Value of $f(x)$ to 5 decimal places		Value of x	Value of $f(x)$ to 5 decimal places
-4.0			0.5	
-3.5			1.0	Neat!
-3.0			1.5	
-2.5			2.0	
-2.0			2.5	
-1.5			3.0	
-1.0			3.5	
-0.5			4.0	
0.0			4.5	

b. Go to **STAT Edit** and enter the values of x in $\mathbf{L_1}$ and the values of $f(x)$ in $\mathbf{L_2}$. Use the **STAT PLOT** feature to graph the function. **Turn off the axes.**







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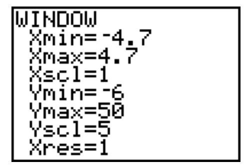
c. Enter the function $g(x) = e^x$ in the **Y=** screen and graph it. (Keep the axes off.) What's the relationship between $f(x)$, the function determined by the series, and $g(x)$?

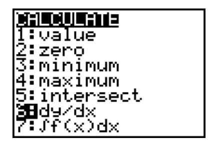
d. Use your graphing calculator's **sum(** and **seq(** features to see if the equation $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^{n-1}}{(n-1)!} + \cdots$ would give calculator manufacturers a good approximation formula for e^x .

4. Enter the function $g(x) = e^x$ in the **Y=** screen and graph it. (Be certain to turn off any **STAT PLOTS** that are on.) Use the window settings shown at right and keep the axes turned off.



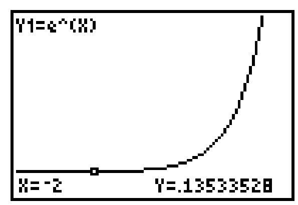
a. The **slope feature**, often denoted *dy*/*dx*, is read as "the change in *y* over the change in *x*." It calculates the slope of the line that is tangent to a given graph at a given point.

You access the slope feature by pressing **2nd TRACE** for **CALCULATE**, arrowing to **6** for **slope**, and pressing **ENTER**. Use this feature to fill in the table below.

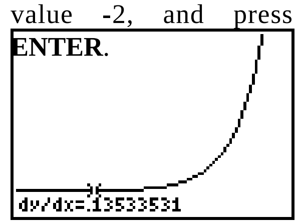


Note: Write down ALL decimal places.

x-value	*y*-value	*dy/dx*
-2.0		
-1.4		
-1.0		
0.0		
0.7		
1.0		
2.3		
3.6		



Press **TRACE**, enter the value -2, and press **ENTER**.



Press **2nd TRACE** for **CALCULATE**, arrow to **6** for **slope**, and press **ENTER** for *dy/dx*.

b. What do you notice about the *y*-value and $\frac{dy}{dx}$?

c. Does this work for any function or is $g(x) = e^x$ a **very** special function?