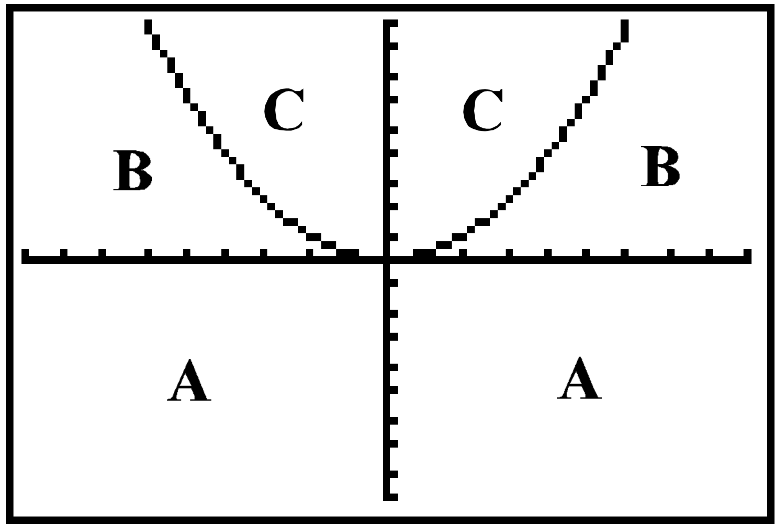
Consider the function $f(x)=x^2+rx+s$. What is the probability that the function will have two real roots if $-9\le r,s\le 9$?

We have already determined that the function will have two real roots when $r^2-4s>0$. Thus, the question becomes: What is the probability that (r, s) lies below the curve $s=\frac{r^2}{4}$?



From An Area Perspective:

P((r, s) lies below the curve) is:

$$=\frac{\text{Area under the curve}}{\text{Total area of the square}}=\frac{2A+2B}{2A+2B+2C}=\frac{2(A+B)}{2(A+B+C)}=\frac{A+B}{A+B+C}$$

$$=\frac{81+\frac{5}{9}(81)}{81+\frac{5}{9}(81)+\frac{4}{9}(81)}=\frac{81\left(1+\frac{5}{9}\right)}{81\left(1+\frac{5}{9}+\frac{4}{9}\right)}=\frac{1+\frac{5}{9}}{1+\frac{5}{9}+\frac{4}{9}}=\frac{\frac{14}{9}}{2}=\frac{14}{18}=.\overline{7}$$

From A Probability Perspective:

Note: Since the probability of a point being selected from one region has no effect on the probability of selecting a point from another region, the outcomes (selections) are independent. Thus, $P(2A\cap 2B)=P(2A)P(2B)$.

P((r, s) lies below the curve) is:

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$$P(2A+2B)=P(2A)+P(2B)-P(2A \cap 2B)$$

$$=\frac{1}{2}+\frac{5}{9}-\left(\frac{1}{2}\right)\left(\frac{5}{9}\right)$$

$$=\frac{14}{18}=.\overline{7}$$