

Time Value of Money Practice Problems – Solutions

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- 1) Jim makes a deposit of \$12,000 in a bank account. The deposit is to earn interest annually at the rate of 9 percent for seven years.

- a) How much will Jim have on deposit at the end of seven years?

$$P/Y = 1, N = 7, I = 9, PV = 12,000, PMT = 0 \Rightarrow FV = \$21,936.47$$

- b) Assuming the deposit earned a 9 percent rate of interest compounded quarterly, how much would he have at the end of seven years?

$$P/Y = 4, N = 7 \times 4 = 28 \Rightarrow FV = \$22,374.54$$

- c) In comparing parts (a) and (b), what are the respective effective annual yields? Which alternative is better?

Because interest is compounded annually in part (a), the effective annual rate is the same as the nominal rate: $EAR_A = 9\%$.

In part (b), $EAR_B = (1 + i/m)^m - 1 = 1.0225^4 - 1 = 9.31\%$.

This can be also solved using the TI BAII+ using the Interest Conversion worksheet. Simply press [2nd] [I Conv] (the second function of the 2 key) to bring up this worksheet. When the screen says NOM = press [9] and [Enter]. Then arrow up and make sure that [C/Y] reads 4 compounding periods per year; if not, press [4] and [Enter]. Finally arrow up to the EFF screen and press [CPT] to compute the effective annual rate.

Alternative (b) is preferred because it compounds your interest more frequently. Thus you get to earn “interest on your interest” sooner.

- 2) John is considering the purchase of a lot. He can buy the lot today and expects the price to rise to \$15,000 at the end of 10 years. He believes that he should earn an investment yield of 10 percent annually on this investment. The asking price for the lot is \$7,000. Should he buy it? What is the annual yield (internal rate of return) of the investment if John purchases the property for \$7,000 and is able to sell it 10 years later for \$15,000?

$P/Y = 1, N = 10, I = 10, PMT = 0, FV = 15,000 \Rightarrow PV = -\$5,783.15$. Because the present value of this investment is less than the \$7,000 asking price for the lot, John should not buy it.

To solve for the internal rate of return enter $PV = -7,000$ and compute $I = 7.92\%$.

- 3) An investor can make an investment in a real estate development and receive an expected cash return of \$45,000 after six years. Based on a careful study of other investment alternatives, she believes that an 18 percent annual return compounded quarterly is a reasonable return to earn on this investment. How much should she pay for it today?

$$P/Y = 4, N = 6 \times 4 = 24, I = 18, PMT = 0, FV = 45,000 \Rightarrow PV = -\$15,646.66$$

- 4) Suppose you have the opportunity to make an investment in a real estate venture that expects to pay investors \$750 at the end of each month for the next eight years. You believe that a reasonable return on your investment should be 17 percent compounded monthly.

- a) How much should you pay for the investment?

$$P/Y = 12, N = 8 \times 12 = 96, I = 17, PMT = 750, FV = 0 \Rightarrow -\$39,222.96.$$

- b) What will be the total sum of cash you will receive over the next eight years?

This can be solved by setting $I = 0$, $PV = 0$, and computing $FV = -\$72,000$. Notice that the sign of this solution is negative because the payments have been entered as positive values.

- c) Why is there such a large difference between (a) and (b)?

The difference between the answers in parts (a) and (b) represents the foregone interest that results from receiving the payments in the future, rather than today.

- 5) Walt is evaluating an investment that will provide the following returns at the end of each of the following years: year 1, \$12,500; year 2, \$10,000; year 3, \$7,500; year 4, \$5,000; year 5, \$2,500; year 6, \$0; and year 7, \$12,500. Walt believes that he should earn an annual rate of 9 percent on this investment. How much should he pay for this investment?

This can be solved using the irregular cash flow worksheet:

$$\begin{aligned}CF_0 &= 0 \\CF_1 &= 12,500 \\CF_2 &= 10,000 \\CF_3 &= 7,500 \\CF_4 &= 5,000 \\CF_5 &= 2,500 \\CF_6 &= 0 \\CF_7 &= 12,500\end{aligned}$$

Set $I = 9$ and solve for $NPV = \$37,681$.

- 6) A loan of \$50,000 is due 10 years from today. The borrower wants to make annual payments at the end of each year into a sinking fund that will earn interest at an annual rate of 10 percent. What will the annual payments have to be?

Suppose that the borrower will make monthly payments that earn 10 percent interest, compounded monthly. How much will he pay annually into the fund?

$$\begin{aligned}\text{With annual compounding: } P/Y &= 1, N = 10, I = 10, PV = 0, FV = 50,000 \\ \Rightarrow PMT &= -\$3,137.27.\end{aligned}$$

$$\begin{aligned}\text{With monthly compounding: } P/Y &= 12, N = 10 \times 12 = 120, \Rightarrow PMT = -244.09. \\ \text{These are monthly payments, so the total annual payment will be} \\ 244.09 \times 12 &= \$2,929.04.\end{aligned}$$

- 7) The Dallas Development Corporation is considering the purchase of an apartment project for \$100,000. They estimate that they will receive \$15,000 at the end of each year for the next 10 years. At the end of the 10th year, the apartment project will be worth nothing. If Dallas purchases the project, what will be its internal rate of return? If the company insists on a 9 percent return compounded annually on its investment, is this a good investment?

$P/Y = 1, N = 10, I = 9, PMT = 15,000, FV = 0 \Rightarrow PV = -\$96,264.87$. Based on the NPV rule, this is a poor investment because the present value of future cash flows is less than the required investment of \$100,000.

Alternatively, you could enter $PV = -\$100,000$ and solve for $I = 8.14\%$. Because the IRR of this investment is less than the 9% hurdle rate, Dallas should not invest in this project.

- 8) Suppose you deposit \$5,000 into an account earning 4 percent interest, compounded monthly.

- a) How many years will it take for your account to be worth \$7,500?

$P/Y = 12, PV = -5,000, I = 4, PMT = 0, FV = 7,500 \Rightarrow N = 121.84$, or 10.15 years.

- b) Suppose in addition to the initial \$5,000 deposit, you will make monthly contributions of \$50. How many years will it take for the account to grow to \$7,500 in this case?

$PMT = -50 \Rightarrow N = 35.39$ or 2.95 years.

- c) How does your answer change if you make quarterly deposits of \$150 rather than monthly contributions of \$50? Explain the reason for any difference in your answer from part b. Maintain the assumption that interest compounds monthly.

$P/Y = 4, C/Y = 12, PMT = -150 \Rightarrow N = 11.83$ or 2.96 years.

The time it takes to save \$7,500 rises just a bit because you are no longer earning interest on new investments until the end of each quarter, rather than each month.

- 9) Consider an investment that will pay \$680 per month for the next 15 years and will be worth \$28,000 at the end of that time. How much is this investment worth to you today at a 5.25 percent discount rate?

$P/Y = 12, N = 15 \times 12 = 180, I = 5.25, PMT = 680, FV = 28,000 \Rightarrow PV = -97,351.34$.

- 10) You currently owe \$18,000 on a car loan at 9.5 percent interest. If you make monthly payments of \$576.59 per month, how long will it take you to fully repay the loan?

$P/Y = 12, I = 9.5, PV = 18,000, PMT = -576.59, FV = 0 \Rightarrow N = 36$

- 11) You have just borrowed \$10,000 and will be required to make monthly payments of \$227.53 for the next five years in order to fully repay the loan. What is the implicit interest rate on this loan?

$$P/Y = 12, N = 5 \times 12 = 60, PV = 10,000, PMT = -227.53, FV = 0 \Rightarrow I = 13\%$$

- 12) Your uncle has given you a bond that will pay \$500 at the end of each year forever into the future. If the market yield on this bond is 8.25 percent, how much is it worth today?

This type of investment is known as a perpetuity. The formula for its value is $PV = PMT / r = 500 / 0.0825 = 6,060.61$.

- 13) Suppose you have an investment that is expected to generate a \$20,000 cash flow next year and that this is expected to increase by 5 percent per year forever into the future.

- a) If your required rate of return on this investment is 18 percent, how much is it worth to you today?

This is a growing perpetuity. The formula for its value is

$$PV = \frac{PMT}{r - g} = \frac{20,000}{0.18 - 0.05} = 153,846.15.$$

- b) Suppose now that you do not know how fast the cash flows will grow in the future, but that you expect them to grow at a constant rate. Suppose also that this investment is currently priced at \$200,000. If the required rate of return is still 18 percent, how fast does the market expect the annual cash flows to grow?

Rearrange the growing perpetuity formula to solve for the growth rate:

$$g = r - \frac{PMT}{PV} = 0.18 - \frac{20,000}{200,000} = 0.18 - 0.10 = 0.08 \text{ or } 8 \text{ percent.}$$

- 14) You are considering the purchase of an investment that is expected to generate cash flows of \$15,000 per year for the next five years. After that, cash flows are expected to increase at the rate of 5 percent per year for the indefinite future. Thus, in year 6 the cash flow will be \$15,750, etc. How much is this investment worth to you today if your required return is 15 percent?

First, calculate the value of the investment at the end of year 5 using the formula for a growing perpetuity:

$$V_5 = \frac{CF_6}{r - g} = \frac{15,750}{0.15 - 0.05} = 157,500.$$

Next use this as the future value and use your time value of money keys to calculate the value as of date 0 (today).

$$P/Y = 1, N = 5, I = 15, PMT = 15,000, FV = 157,500 \Rightarrow PV = -128,587.66.$$

- _____ 1. What is the future value of \$3,500 deposited for 12 years at 5 percent interest, compounded annually?
- A. \$6,285.50
 B. \$3,679.07
 C. \$55,709.94
 D. \$6,369.47

P/Y = 1, N = 12, I = 5, PV = 3,500, PMT = 0; solve for FV

- _____ 2. What is the internal rate of return on an investment that costs \$2,000 and returns \$32 per month for the next 15 years?
- A. The IRR cannot be calculated for this investment
 B. 1.48%
 C. 17.86%
 D. 14.13%

P/Y = 12, PMT = 32, PV = -2,000, FV = 0, N = 15 × 12 = 180; solve for I

- _____ 3. What is the internal rate of return of an investment with the following cash flows?

n	\$
0	(1,000)
1	300
2	300
3	300
4	200
5	100

- A. The IRR cannot be calculated for this investment
 B. 10.00%
 C. 15.23%
 D. 7.48%

CF0 = -1,000, C01 = 300, F01 = 3, C02 = 200, F02 = 1, C03 = 100, F03 = 1; solve for IRR

- _____ 4. If your discount rate is 12%, what is the NPV of the investment from the last question?
- A. \$0.00
 B. (\$55.25)
 C. \$200.00
 D. (\$95.60)

With all the same entries, solve for NPV with I = 12

- _____ 5. True or ***FALSE***: Assuming the same positive discount rate and the same number of years over which they will be received, a \$1,200 annuity with annual payments has a larger present value than a \$100 annuity with monthly payments.

The reason is “sooner is better than later.” The total payments you receive is the same under both annuities (\$1,200 per year), but you get some of your money sooner with the monthly annuity. You could put this in an account and earn interest on it, making it worth more to you than \$1,200 at the end of each year.

- _____ 6. How much should you pay for an investment that pays \$1,500 per year for the next three years and then \$2,000 per year for the following two years? Assume a discount rate of 15% per year?
- A. \$8,500.00
 - B. \$5,562.70***
 - C. \$6,338.15
 - D. \$10,000.00

$CF_0 = 0$, $C_01 = 1,500$, $F_01 = 3$, $C_02 = 2,000$, $F_02 = 2$; solve for NPV with $I = 15$

- _____ 7. Consider an eight-year investment costing \$55,000. It is expected to pay \$3,000 per year in each of the next five years and \$15,000 per year in the last three years. If the required discount rate is 12 percent, what is the net present value of this investment?
- A. \$86,257.28
 - B. (\$23,742.72)***
 - C. \$5,000
 - D. \$60,000.00

$CF_0 = -55,000$, $C_01 = 3,000$, $F_01 = 5$, $C_02 = 15,000$, $F_02 = 3$; solve for NPV with $I = 12$

- _____ 8. What is the internal rate of return of the investment in the last question?
- A. The IRR cannot be calculated for this investment
 - B. 1.47%***
 - C. 14.7%
 - D. 12.0%

With the same entries as above, solve for IRR

- _____ 9. What is the *annual debt service* on a \$1.2 million 30-year mortgage at 6.75% interest with monthly payments?
- A. \$40,518
 - B. \$7,783
 - C. \$94,286.59
 - D. **\$93,398**

Annual debt service is simply 12 times the monthly payment. Enter PV = 1,200,000, P/Y = 12, N = $30 \times 12 = 360$, I = 6.75, FV = 0; solve for PMT and multiply the result by 12.

- _____ 10. In class we said that “cash is king” in finance. Which of the following things is NOT true about this king?
- A. More is better than less
 - B. Sooner is better than later
 - C. Certain is better than uncertain
 - D. **STRAIGHT IS BETTER THAN CROOKED**