The shot noise of photons is a big barrier in a range of applications like the coherent processing of signals, quantum communication and quantum metrology. Use of squeezed light would be a way out and great progress has been made towards producing light with large squeezing. In this talk I would discuss a newer possibility i.e. the use of sources of light which produce entangled photon pairs. I would describe the progress made using entangled photon pairs and the challenges ahead.

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Application Areas

- Using entangled photon sources like Parametric Down Conversion (PDC) to produce superresolution and supersensitivity
- Magneto-optical Rotation (MOR) of light and sensing of magnetic fields
- Measurement of rotations as in Sagnac Interferometers, gyroscopes
- Sensitive measurement of refractive index
- Improving the sensitivity of light scattering measurements
- Quantum sensors, target detection
Quantum Sensing with Nonclassical and Entangled Light

Quantum Noise of Measurement: \( \Delta \phi \cdot \Delta N \sim 1 \)

Coherent Sources: \( \Delta N \sim \sqrt{N} \)

Traditional Measurements: \( \Delta \phi \sim \frac{1}{\sqrt{N}} \)

Heisenberg Limited Measurements: \( \Delta \phi \sim \frac{1}{N} \)
Quantum Sensing with Nonclassical and Entangled Light

- Two single photons $|v\rangle|v\rangle$

- Entangled state (uncertainty in direction)
  \[
  \frac{1}{\sqrt{2}}(|v\rangle|v\rangle + |h\rangle|h\rangle)
  \]

- Classical probability theory
  \[
  \frac{1}{\sqrt{2}}(|v\rangle|v\rangle\langle v|\langle v| + |h\rangle|h\rangle\langle h|\langle h|)
  \]
Quantum Entanglement

First introduced by E. Schrödinger
[Naturwissenschaften, 23, 807 (1935)]

To explain intriguing features of a composite system

“Any state of a composite system is said to be entangled if it cannot be expressed as a direct product of the state of each subsystem.”

\[
\Psi_{AB} = \Phi_A \otimes \chi_B \equiv \text{Not entangled}
\]

\[
\Psi_{AB} = \sum_{ij} C_{ij} \Phi_i^A \otimes \chi_i^B \equiv \text{Entangled} \quad \text{if } C_{ij} \neq C_i C_j
\]

Basic bipartite entangled states:

\[
\Psi^{\pm}_{AB} = \frac{1}{\sqrt{2}} \left( |0\rangle_A |1\rangle_B \pm |1\rangle_A |0\rangle_B \right)
\]

\[
\Phi^{\pm}_{AB} = \frac{1}{\sqrt{2}} \left( |0\rangle_A |0\rangle_B \pm |1\rangle_A |1\rangle_B \right)
\]

Spin ↑↓; Polarization - right, left circular/x, y; BEC;
PDC as a Source of Entanglement

\[ \vec{k}_0, \omega_0 \rightarrow \vec{k}_1, \omega_1 \]

\[ \vec{k}_2, \omega_2 \]

\[ \vec{k}_0 = \vec{k}_1 + \vec{k}_2, \quad \omega_0 = \omega_1 + \omega_2 \]

\[ H = \hbar (g a_0 a_1^+ a_2^+ + h.c.); \quad g \propto \chi^{(2)} \]

( Sum over all k’s; frequencies ; Polarizations etc.)

\[ a_0: \quad \text{Coherent pump} \]

\[ H \begin{vmatrix} \alpha_0, 0, 0 \end{vmatrix} \rightarrow \begin{vmatrix} \alpha_0, 1, 1 \end{vmatrix} \]

\[ |\psi \rangle \rightarrow \mu |\alpha_0, 0, 0 \rangle + \nu |\alpha_0, 1, 1 \rangle \]
Types of PDC

Type-II collinear

$$|\psi\rangle = \frac{1}{\cosh g} \sum_{n=0}^{\infty} (e^{i\theta} \tanh g)^n |n\rangle_H |n\rangle_V$$

Type-II non-collinear

$$|\psi\rangle = \frac{1}{\cosh^2 g} \sum_{n=0}^{\infty} \sum_{m=0}^{n} (e^{i\theta})^m (\tanh g)^n |n-m\rangle_{a_H} |m\rangle_{a_V} |m\rangle_{b_H} |n-m\rangle_{b_V}$$
Entangled Photon Pairs

**ENTANGLED PHOTON PAIRS** are created when a laser beam passes through a crystal such as beta barium borate. The crystal occasionally converts a single ultraviolet photon into two photons of lower energy, one polarized vertically (on red cone), one polarized horizontally (on blue cone). If the photons happen to travel along the cone intersections (green), neither photon has a definite polarization, but their relative polarizations are complementary; they are then entangled. Colorized image (at right) is a photograph of down-converted light. Colors do not represent the color of the light.
Operation: \( U = e^{\frac{i\pi S_x}{2}} \)

\[
\left| \downarrow\downarrow\downarrow\cdots\downarrow \right\rangle \xrightarrow{U} \text{Entangled State: CAT}
\]

\[
|\Psi\rangle = U\left| \downarrow\downarrow\downarrow\cdots\downarrow \right\rangle = e^{i\pi/4} \frac{1}{\sqrt{2}} \left( \left| \downarrow\downarrow\downarrow\cdots\downarrow \right\rangle + i^N e^{-\frac{i\pi}{2}} \left| \uparrow\uparrow\uparrow\cdots\uparrow \right\rangle \right)
\]
Heisenberg Limited Measurements

\[ U = e^{\frac{i\pi S_x^2}{2}} \]

\[ |\Psi_{\text{out}}\rangle = \exp\left[ -i \frac{\pi}{2} S_x^2 \right] e^{i\phi S_z^z} |\text{CAT}\rangle = \cos\left( \frac{N\phi}{2} \right) |\downarrow\downarrow\cdots\downarrow\rangle + (i)^N \sin\left( \frac{N\phi}{2} \right) |\uparrow\uparrow\cdots\uparrow\rangle \]
Signal \[ \langle S_z \rangle = \frac{N}{2} \cos(N\phi) \]

Noise \[ (Noise)^2 = \frac{N^2}{4} \sin^2(N\phi) \]

Accuracy of measurement
\[ \frac{(Noise)^2}{(\partial \langle S_z \rangle / \partial \phi)^2} = \frac{N^2}{4} \frac{\sin^2(N\phi)}{N^2 \sin^2(N\phi)} \sim \frac{1}{N^2} \]
\[ \Delta\phi \sim \frac{1}{N} \]
Entangled Photon Pairs from PDC

Input state: \[ |\psi\rangle = \frac{1}{\cosh g} \sum_{n=0}^{\infty} (-e^{i\theta} \tanh g)^n |n\rangle_1 |n\rangle_2 \]

\[ P_2 = |\langle 11 | U | \psi \rangle|^2 = \frac{\tanh^2 g}{2 \cosh^2 g} [1 + \cos(2\phi)] \]
Can We Improve Resolution Further?

Multi-Photon Entanglement!

\[
P_4 = \frac{\tanh^4 g}{\cosh^2 g} T_1 T_2 R_1 R_2 \frac{1}{8} [11 + 12 \cos(2\phi) + 9 \cos(4\phi)]
\]
Can We Get Fringes with a Constant Contrast?

\[
P_4 = \frac{\tanh^4 g}{\cosh^2 g} \frac{9}{8} T_1^2 R_1 R_2 \left[1 - \cos(4\phi)\right]
\]

How does this work?

All interferometric schemes using the maximally correlated entangled states show the phase sensitivity approaching to 1/N, which is the Heisenberg limit.

**Classical field**

\[ |\alpha\rangle \rightarrow |\alpha e^{i\phi}\rangle \]

**Phase shifter**

\[ |n\rangle \rightarrow e^{i\frac{\sqrt{4}n^2}{|n\rangle}} |n\rangle \]

**Non-classical Fock state**

\[ |11\rangle \rightarrow \frac{|02\rangle + e^{i2\phi} |02\rangle}{\sqrt{2}} \]

after the first BS and Phase shifter

\[ |22\rangle \rightarrow \frac{\sqrt{3}}{4} \left( \frac{3}{|00\rangle + e^{i4\phi} |04\rangle}{\sqrt{2}} \right) + \frac{1}{\sqrt{4}} e^{i2\phi} |22\rangle \]

after the first BS and Phase shifter

\[ \rightarrow \frac{\sqrt{6}}{4} \sin^2 \phi [|00\rangle + |04\rangle] + \frac{\sqrt{6}}{4} \sin 2\phi [|31\rangle - |13\rangle] + (2 - 3\sin^2 \phi) |22\rangle \]

after the second BS

**Non-classical Fock state (continued)**

\[ \frac{\sin \phi}{\sqrt{2}} [|02\rangle - |02\rangle + \cos \phi |11\rangle] \]

after the second BS

**Only this part contributes to the detection in the setup given by Fig. 6**

**This term leads to unequal fringes in the detection setup given by Fig. 5**
Beating the Standard Quantum Limit with Four-Entangled Photons

Fig. 1. An optical interferometer for beating the SQL. (Inset) A schematic of a Mach-Zehnder (MZ) interferometer consisting of two 50:50 beam splitters (BS1 and BS2). Photons are input in modes $a$ and/or $b$, and detected in modes $e$ and/or $f$, after a phase shift (PS) is applied to mode $d$. (Main panel) A schematic of the intrinsically stable displaced-Sagnac architecture used to ensure that the optical path lengths in modes $c$ and $d$ are subwavelength (nm) stable. A frequency-doubled 780-nm fs-pulsed laser (repetition interval 13 ns) pumps a type I phase-matched beta barium borate (BBO) crystal to generate the state $|22\rangle_{ab}$ via spontaneous parametric down-conversion. Interference filters (not shown) with a 4-nm bandwidth were used. The photons are guided via polarization-maintaining fibers (PMFs) to the interferometer, which has the same function as the MZ interferometer in the inset. A variable phase shift in mode $d$ is realized by changing the angle of the phase plate (PP) in the interferometer. Photons are collected in single-mode fibers (SMFs) at the output modes and detected with a single-photon counting module (SPCM, detection efficiency 60% at 780 nm) in mode $f$ and three cascaded SPCMs in mode $e$.

Fig. 3. Beating the SQL with four-entangled photons. (A) Single-photon count rate in mode $e$ as a function of phase plate (PP) angle with single-photon input $|10>_{ab}$. (B) Two-photon count rate in modes $e$ and $f$ for input state $|11>_{ab}$. (C) Four-photon count rate of three photons in mode $e$ and one photon in mode $f$ for the input state $|22>_{ab}$. Accumulation times for one data point were (A) 1s, (B) 300s, and (C) 300s.
Beating the Standard Quantum Limit – High Gain OPA

\[ |\psi\rangle_{a_1b_1} = \frac{1}{\cosh g} \sum_n \left( e^{i\theta} \tanh g \right)^n |n,n\rangle_{a_1b_1} \]

mean photon number per mode: \( \sim \sinh^2 g \)

In the present paper we experimentally demonstrate that the output of a high-gain optical parametric amplifier can be intense yet exhibits quantum features, namely, sub-Rayleigh fringes, as proposed by [Agarwal et al., Phys. Rev. Lett. 86, 1389 (2001)]. We investigate multiphoton states generated by a high-gain optical parametric amplifier operating with a quantum vacuum input for gain values up to 2.5.

Large gain – How to improve visibility

\[
\begin{pmatrix}
\hat{a}_3 \\
\hat{b}_3
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \left( \mu(\hat{a}_0 + \alpha) + \nu(\hat{b}_0^\dagger + \alpha^*) \right)
\]

\[
\mu = \cosh g \quad g, \phi, \psi : \text{Interaction Parameter, Pump Phase, Interferometric Phase}
\]

\[
\nu = e^{i\psi} \sinh g \quad \alpha = |\alpha| e^{i\theta} : \text{coherent beam amplitude}
\]
Mean intensity and two-photon counts

\[ I_{a_3} \equiv \langle \hat{a}_3 \dagger \hat{a}_3 \rangle = \sinh^2 g + |\alpha|^2 \left[ \cosh^2 g + \sinh^2 g + 2 \sinh^2 g \cosh^2 g \cos(\psi - 2\theta) \right] \left[ 1 - \sin(\phi) \right] \]

\[ I_{b_3} \equiv \langle \hat{b}_3 \dagger \hat{b}_3 \rangle = \sinh^2(g) + |\alpha|^2 \left[ \cosh^2 g + \sinh^2 g + 2 \sinh^2 g \cosh^2 g \cos(\psi - 2\theta) \right] \left[ 1 + \sin(\phi) \right] \]

\[ \psi, \theta, \phi: \] pump phase, coh. beam phase, interferometer phase

\[ \langle \hat{a}_3 \dagger \hat{b}_3 \dagger \hat{b}_3 \hat{a}_3 \rangle \]

two-photon coincidence counts

visibility of \[ \langle \hat{a}_3 \dagger \hat{b}_3 \dagger \hat{b}_3 \hat{a}_3 \rangle \]
dashed line: coherent beam off
solid line: coherent beam on with phase at \( \pi/2 \)
g: the interaction (squeezing) parameter
(a) Stimulated emission enhanced two-photon counts for various phases of the coherent field at the gain $g = 0.5$. The horizontal line shows the interferometric phase. The pump phase $\psi$ is fixed at $\pi$. The counts are in units of two-photon coincidence rates coming from spontaneous down-conversion process. The modulus of the coherent $|\alpha|$ is chosen such that the coincidences coming from SPDC and the coherent fields are equal to each other. The dashed line shows the two-photon counts for the case of spontaneous process. (b) The same with (a) at the gain $g = 2.0$. Here, the counts for the case of spontaneous process (dashed line) is multiplied by a factor of $10^3$.

A New Class of Interferometers

SU[1,1] Interferometers

A strong laser beam pumps the first OPA. The beam (which is assumed to be undepleted) undergoes a $\pi$-phase shift and then pumps the second OPA. The input modes of the first OPA are fed with coherent light. After the first OPA, one of the outputs interacts with the phase to be probed. Both outputs are then brought back together as the inputs for the second OPA. Measurements are taken on the second OPA's outputs.
SU[1,1] Interferometers

\[
\Delta \phi^2 = \frac{\langle \hat{N}_T^2 \rangle - \langle \hat{N}_T \rangle^2}{\left( \frac{\partial \langle \hat{N}_T \rangle}{\partial \phi} \right)^2}
\]

\[
\hat{N}_T = \hat{a}_f^+ \hat{a}_f + \hat{b}_f^+ \hat{b}_f
\]

When \( \phi = 0 \) and \( \theta = \pi/4 \) (the phase of the input coherent states (which are taken to be equal)),

\[
\Delta \phi^2 = \frac{1}{N_{\text{OPA}} (N_{\text{OPA}} + 2)} \frac{1}{N_{\text{Coh}}}
\]

Where \( N_{\text{OPA}} = 2 \sinh^2(r) \), \( N_{\text{Coh}} = |\alpha|^2 + |\beta|^2 \), \( |\alpha| = |\beta| \).
Lidar with entangled photons based on Mie scattering

Classical light scattering particle

Back scattering generally the polarization changes

Resolution: scatterer Fourier components $\leq \frac{4\pi}{\lambda}$

(Born & Wolf) p. 702

Two detectors

Measure cross correlations; detectors are set to measure certain polarizations

Cross correlations expected to yield better resolution.
**Magneto-optical (Faraday) Rotation (MOR)**

\[ \varepsilon_y = \frac{-i}{\sqrt{2}} (\varepsilon_+ - \varepsilon_-) \]

\[ \phi = k l (\chi_+ - \chi_-) \]

\[ \phi \propto B \text{ for weak fields} \]
Joint Probability of Detecting $N$ Photons and $M$ photons at Different Ports Parity Measurements

Parity detection is the measurement of the evenness or oddness of the number of photons in an optical mode.

It is represented by the operator:

\[ \hat{\Pi} = (-1)^N \]

where \( N \) is the number operator.

Parity detection has been shown to perform very well across a wide variety of input states to an MZI.
Super Resolution via Newer Observables

- The power of parity measurement is highlighted in a TMSV input MZI.

- In this setup parity detection reaches slightly below the Heisenberg limit on phase sensitivity.

\[
\langle \hat{\Pi}_A \rangle_{\varphi+\pi/2} = \langle \hat{\mu}_{AB} \rangle_{\varphi} = \frac{1}{\sqrt{1 + \bar{n}(\bar{n} + 2)\sin^2 \varphi}},
\]

Super Resolution via Newer Observables

- However practical measurement of the parity operator is non-trivial.

- Parity may be simulated with number-resolving detectors, but this is more than is necessary.

- Furthermore number-resolving detectors are difficult to build and operate, and are only accurate at small numbers of photons.

- However it can be shown that the Wigner function at the origin is equivalent to parity.

- This can be seen easily in the case of number states.

a) n=0, b) n=1 c) n=5 Img. By J S Lundeen
Mathematically the statement is

\[ \langle \hat{\Pi} \rangle = \frac{\pi}{2} W(0, 0). \]

If we restrict ourselves to states with Gaussian Wigner distributions we can write in general

\[
W(\alpha, \alpha^*) = \frac{1}{\pi \sqrt{\tau^2 - 4|u|^2}} e^{-\frac{u(\alpha - \alpha_o)^2 + u^*(\alpha - \alpha_o)^*^2 + \tau |\alpha - \alpha_o|^2}{\tau^2 - 4|u|^2}}
\]

\[ W(0, 0) = \frac{1}{\pi \sqrt{\tau^2 - 4|u|^2}}. \]

where \( \langle \hat{a} \rangle = \alpha_o, \langle \hat{a}^\dagger 2 \rangle - \langle \hat{a}^\dagger \rangle^2 = -2u, \) and \( \langle \hat{a}^\dagger \hat{a} + \frac{1}{2} \rangle - \langle \hat{a}^\dagger \rangle \langle \hat{a} \rangle = \tau \)
Question: Given $N$ photons with orbital angular momentum (OAM) $lh$ per photon, how accurately can angular-displacements be measured?

Dove prism mode transformation:

$$|l\rangle \rightarrow e^{i\pi} e^{-2il\theta} | - l\rangle$$

($\theta$ is the angle of rotation)

Achievable Angular sensitivity:

$$\Delta \theta = \frac{1}{2\sqrt{Nl}}$$

Supersensitive Angular-Displacement Measurements

Question: Given $N$ photons with orbital angular momentum (OAM) $l\hbar$ per photon, how accurately can angular-displacements be measured?

$N$ One-Photon Fock-State Input

Dove prism mode transformation:

$$|l\rangle \rightarrow e^{i\pi} e^{-2i\theta} |l\rangle - l\rangle (\theta \text{ is the angle of rotation})$$

Achievable Angular sensitivity:

$$\Delta \theta = \frac{1}{2\sqrt{Nl}}$$

Two-Photon Entangled-State Input

Input State: $|\psi_2\rangle = \sqrt{\frac{1}{2}} [|l\rangle_s - l\rangle_i + | - l\rangle_s |l\rangle_i]$}

Post-selected two-photon state in modes $a$ and $b$:

$$|\psi_2^l\rangle_{ab} = \frac{1}{4} [(1 + e^{-4il\theta})|l\rangle_a - l\rangle_b]$$

Angular Sensitivity:

$$\Delta \theta = \frac{\langle \Delta \hat{A} \rangle}{|\partial \langle \hat{A} \rangle / \partial \theta|} = \frac{1}{4l}$$
Supersensitive Angular-Displacement Measurements

Four-Photon Entangled-State Input

Input State: \( |\psi^I_4\rangle = \frac{1}{2} \left[ |l, l\rangle_s - l, -l\rangle_i + | -l, -l\rangle_s |l, l\rangle_i + |l, -l\rangle_s |l, -l\rangle_i + | -l, l\rangle_s |l, -l\rangle_i \right] \)

Post-selected four-photon state in modes \( a \) and \( b \): \( |\psi^I_4\rangle_{ab} \propto \left[ 1 - e^{-i2l\theta} \right] |l, l\rangle_a |l, l\rangle_b \)

The Measurement Operator: \( \hat{A} = |l, l, l\rangle_a - l, b \rangle_a |l, l, l\rangle_b \)

Angular Sensitivity:
\[ \Delta \theta = \frac{\langle \Delta \hat{A} \rangle}{|\partial \langle \hat{A} \rangle / \partial \theta|} = \frac{1}{8l} \]

N-Photon Entangled-State Input

Suitable Measurement Scheme

Achievable Angular Sensitivity:
\[ \Delta \theta = \frac{1}{2Nl} \]
Novel Ramsey Spectroscopy
Employing Joint-Detection
and Nonclassical Fields

An atom in |g⟩ passing through a field $H_{\text{eff}} \approx \hbar \Delta S_z - \hbar(gS_+ + g^* S_-)$

Probability of getting excited $p_e(\tau) = |g|^2 \tau^2 \text{sinc}^2 X \quad X = \frac{\Delta \tau}{2}$

We find the interference fringe $\tau \rightarrow \text{larger} \rightarrow \text{narrower}$.

The frequency resolution is limited by the time passing through the field.

How can we enhance the resolution by several orders?
Ramsey Spectroscopy – Separate Fields

An atom in $|g\rangle$ passing through two separate fields, and getting excited

$$\rho_R(T + 2\tau) = |g|^2 \tau^2 \sin^2(X) \cdot 4 \cos^2\left(\frac{T}{\tau}X\right)$$

$T \gg \tau$

$T \gg \tau$ → Ramsey’s fringes are sharper than traditional ones.
Ramsey Spectroscopy – Two Path Interferometer

Reason: $|g\rangle + \frac{g}{\Delta}(e^{i\Delta T} - 1)|e\rangle$

Two amplitudes are summed up coherently.
Ramsey Spectroscopy – Go Further

Other possible improvement:

- Stimulated PDC

\[ p_e = G \langle A^\dagger A \rangle = 2G|v|^2 \]

\[ p_{ee} = G^2 \langle A^\dagger A^\dagger AA \rangle = 4G^2 (u^2|v|^2 \cos^2 \Delta T + 2|v|^4). \]

In the squeezed case, fringe oscillates twice as fast.
Conclusions

• Using entangled photon sources like Parametric Down Conversion (PDC) to produce superresolution and supersensitivity

• Magneto-optical Rotation (MOR) of light and sensing of magnetic fields

• Measurement of rotations as in Sagnac Interferometers; gyroscopes

• Improving the sensitivity of light scattering measurements

• Quantum sensors; target detection

• New types of Interferometers

• Newer Detection Schmes

• Atomic Interferometers
MACH-ZEHNDER Interferometer

\[ \Delta l : \text{path difference} \]

Outputs:
\[ \cos^2 \frac{k \Delta l}{2}, \quad \sin^2 \frac{k \Delta l}{2} \]

Rayleigh criterion:
\[ \frac{k \Delta l}{2} = \frac{\pi}{2} \quad \Delta l \leq \frac{\lambda}{2} \]

Super-resolution:
\[ \cos^2 \frac{N \phi}{2}, \quad \phi = k \Delta l \]